Department of Mathematics and Statistics
620–123: Applied Mathematics (Advanced) 2006
Assignment 1

Your completed assignment should be posted by 5pm on Friday August 18 in your
tutor’s assignment box (Richard Berry Building).
You will also be required to hand-in a signed plagiarism declaration (which may be
downloaded from the 123 webpage).

You should give complete explanations in all questions.

1. Compute
\[ 4 \int_0^4 \frac{dx}{(x-2)(x+2)}. \]

2. Compute the indefinite integral
\[ 2 \int \sqrt{1+u^2} \, du. \]

You are allowed to use without proof that \((\text{arcsinh} x)' = \frac{1}{\sqrt{1+x^2}}\) and
\(\sinh(2\text{arcsinh} x) = 2x\sqrt{1+x^2}\).

3. Compute the arc length of the closed curve
\[ \mathbf{r}(t) = (\cos t, \sin^2 t), \quad t \in [0, 2\pi]. \]

4. Solve the IVP
\[ (x+y)y' = x - \frac{y^2}{x}, \quad y(1) = 1. \]

5. Solve the ODE
\[ x^2(1-x)yy' = (y-1)(x+2). \]

6. (a) Show that
\[ y_p(x) = \frac{1}{x} \]

is a particular solution to the ODE
\[ y' = -\frac{1}{x^2} - \frac{y}{x} + y^2. \]

(b) Use the substitution
\[ y = \frac{1}{x} + \frac{1}{v} \]

(with \(v = v(x)\)) to find the general solution.