1. A (very large) population of rabbits and foxes is living together in a forest. Let \( R(t) \) and \( F(t) \) be the number of rabbits and foxes at time \( t \). Since foxes enjoy meeting rabbits but not vice versa, we can model the population by the system of equations

\[
\begin{align*}
\frac{dR}{dt} &= aR - bRF \\
\frac{dF}{dt} &= cF + dRF
\end{align*}
\]

where \( a, b, c, d > 0 \).

(a) Explain the meaning of each of the terms on the right-hand sides.

(b) Treat \( R \) as a function of \( F \) and find an ODE for \( R(F) \).

(c) Solve the ODE.

2. (a) Using a step size of \( h = 1 \), numerically solve the initial value problem

\[
y' = 2x + y - x^2, \quad y(0) = 0, \quad 0 \leq x \leq 2
\]

using the modified Euler method.

(b) Derive the exact solution and use this to determine the errors in your approximations.

3. Find all solutions to the ODE

\[
yy'' = (y')^2 + ay'
\]

where \( a \) is an arbitrary real parameter.

4. Solve the ODE

\[
y'' - 3y' + 2y = 4e^{-x} \cos(2x).
\]

5. Solve the ODE

\[
(1 + x^2)y'' - 2xy' + 2y = (1 + x^2)^2.
\]

Hint: You have to “guess” a solution to the homogeneous ODE.