

Department of Mathematics and Statistics
620-233: Vector Analysis (Advanced) 2008
Assignment 4

This assignment should be returned by 12 noon on Monday May 12.

You should give complete explanations in all questions.

1. (a) Change the order of integration and evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y) dy dx$.
(b) Rewrite the integral $\int_0^1 \int_0^x \int_0^y f(x, y, z) dz dy dx$ to be in the order $dx dy dz$.
2. Let R be the region in the xy -plane lying outside the circle $x^2 + y^2 = 1$ and inside the circle $x^2 + y^2 = 2x$.
 - (a) Sketch the region and describe it in terms of polar coordinates.
 - (b) Find the centre of mass of this region, assuming that it has density $\mu(x, y) = 1/\sqrt{x^2 + y^2}$.
3. Evaluate $\iint_R xy^2 dA$, where R is the region in the first quadrant bounded by the curves $xy = 1$, $xy = 4$, $y = x$, and $y = 2x$ by making a change of variables: $u = xy$, $v = y/x$.
4. (a) Calculate $\iiint_{A_R} \frac{dV}{(x^2 + y^2 + z^2)^c}$ where $A_R = \{(x, y, z) \in \mathbb{R}^3 \mid 1 \leq x^2 + y^2 + z^2 \leq R^2\}$.
(b) Let $A = \{(x, y, z) \in \mathbb{R}^3 \mid 1 \leq x^2 + y^2 + z^2\}$. For what values of c does the improper integral $\iiint_A \frac{dV}{(x^2 + y^2 + z^2)^c}$ exist? Compute the integral for these values of c .
5. Let W be the solid torus obtained by rotating the disc $D = \{(x-a)^2 + z^2 \leq b^2\}$ in the xz -plane about the z -axis, where $0 < b < a$. This can be parametrized by coordinates (r, θ, ϕ) such that $x = (a + r \cos \phi) \cos \theta$, $y = (a + r \cos \phi) \sin \theta$, $z = r \sin \phi$ where $0 \leq r \leq b$, $0 \leq \theta \leq 2\pi$, $0 \leq \phi \leq 2\pi$.
 - (a) Draw a sketch of W indicating the geometric meaning of r , θ , and ϕ .
 - (b) Find the scale factors h_r , h_θ , and h_ϕ for this coordinate system.
 - (c) Verify that the system is orthogonal.
 - (d) Find the volume of W .
 - (e) A fluid circulating within the solid torus W has velocity vector field $\mathbf{F} = f(r, \theta, \phi)\hat{\mathbf{u}}_\theta$. Compute the divergence and the curl of \mathbf{F} .