

**Tutorial 1 - Revision/Basics**

Some basic manipulation tools that should be familiar from first year (*or re-learned very quickly!*):

$$|z_1 z_2| = |z_1| |z_2| \quad \left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|} \quad \overline{z_1 z_2} = \bar{z}_1 \bar{z}_2 \quad \overline{\left( \frac{z_1}{z_2} \right)} = \frac{\bar{z}_1}{\bar{z}_2} \quad |\bar{z}| = |z|$$

$$\frac{1}{x + iy} = \frac{x - iy}{(x + iy)(x - iy)} = \frac{1}{x^2 + y^2} (x - iy)$$

and in particular

$$\frac{1}{i} = -i.$$

1. Simplify the following, giving final answer in cartesian form:

- (a)  $\frac{2-3i}{4-i}$
- (b)  $\frac{i^4+i^9+i^{16}}{2-i^5+i^{10}-i^{15}}$
- (c) Workbook (WB) Qn. 1.(e)
- (d) WB Qn 3.(b)
- (e) WB Qn 3.(e)
- (f) WB Qn 3.(g)

The argument of a complex number  $z = x + iy$  is a multivalued “function”, denoted  $\arg(z)$ . The *principal value* of the argument is a function where we choose the argument value lying in the (real) interval  $(-\pi, \pi]$ . For this we write  $\text{Arg}(z)$ .

$$\text{Arg}(z) = \arctan(y/x) + k\pi, \quad k \in \{0, \pm 1\}.$$

Where  $k$  is chosen to reflect the quadrant of the complex plane in which  $z$  lies. Recall also that for  $a \in \mathbb{R}$ ,

$$\arctan(-a) = -\arctan(a).$$

2. Find all values of the following:

- (a)  $\arg(2\sqrt{3} - 2i)$
- (b)  $\text{Arg}(-1 + i)$
- (c)  $\text{Arg}((7 - 2i) + (-15 + \sqrt{3}i))$

Basic equation solving ideas are as they would normally be for real numbers. In particular the quadratic formula carries over.

Sometimes it is necessary to write the unknown as  $x + iy$  and then equate real and imaginary parts in order to solve.

If  $p, q \in \mathbb{Z}$ , with  $q \neq 0$  and  $a \in \mathbb{C} \setminus \{0\}$ , then an equation of the form

$$z = a^{p/q}$$

has  $q$  distinct solutions which are found by writing  $a$  in polar form and using the multivalued nature of the argument.

3. Solve the following for  $z$

(a)  $(3 + 4i)^2 - 2\bar{z} = z$

(b)  $z^3 = -1 + i$  (Keep these answers they will be useful in (c).)

(c)  $z^6 + 2z^3 + 2 = 0$

(d) WB Qn 10(e). Give answers in cartesian form.

When asked to prove an equation in general, try to start with one side and manipulate it with direct calculation into the other side. Alternatively manipulate as far as you can see how to, then work on the other side separately, attempting to reach the same point.

NEVER write the equation down and manipulate both sides concurrently. If you are not specifically asked to, DO NOT give an example of the equation being true.

Also be sure to define any new symbols that you introduce (indeed this should happen for any question!).

4. WB Qn 4.(a)