1. $\sqrt{3} - i = 2e^{-i(\pi/6 + 2\pi k)}$, $k \in \mathbb{Z}$

So to solve we consider, $z^{2/3} = (\sqrt{3} - i) = 2e^{-i(\pi/6 + 2\pi k)}$, $k = 0, 1$

$\Rightarrow \quad z = 2^{3/2}e^{-i(3/2)(\pi/6 + 2\pi k)}$

$= 2\sqrt{2}e^{-i\pi/4}, 2\sqrt{2}e^{-i3\pi/4}$

$= 2\sqrt{2}\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right), 2\sqrt{2}\left(-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$

$= 2(1 - i), 2(-1 + i)$

2. (c) i.

ii. $S$ is not open, as $3i \in S$, but there is no open disk around $3i$ contained within $S$. Every open disk around $3i$ will contain both points of $S$ and points not in $S$.

iii. $S$ is not closed, because $C \setminus S$ is not open. This is because $-2i \in C \setminus S$, but every open disk around $-2i$ will contain points of $S$.

3. (a) $|z - a| = r$ is the set of points that are distance $r$ from the point $a$.

$|z - a| + |z - b| = d$ is the set of points with the property have the sum of their distance from point $a$ and their distance from point $b$ is equal to $d$.

(b)
iii. For this one we need some calculation to help see the picture. Let $z = x + iy$. Then

$$\sqrt{(x - 2)^2 + y^2} = 2\sqrt{(x + 1)^2 + y^2}$$

$$\Rightarrow (x - 2)^2 + y^2 = 4(x + 1)^2 + 4y^2$$

$$\Rightarrow x^2 - 4x + 4 + y^2 = 4x^2 + 8x + 4 + y^2$$

$$\Rightarrow 3x^2 + 12x + 3y^2 = 0$$

$$\Rightarrow x^2 + 2 + y^2 = 4$$

So the set is a circle of radius 2 centred at $z = -2$.

Sets (i) and (iii) are closed, (ii) is open and (iv) is neither (think about trying to form an open disk around $z = 0$, which belongs to the complement).

4. (a) Let $z = x + iy$. Then

$$f(z) = \frac{2z^2 + 3}{|z - 1|} = \frac{2(x + iy)^2 + 3}{\sqrt{(x - 1)^2 + y^2}} = \frac{2x^2 - 2y^2 + 3 + i(4xy)}{\sqrt{(x - 1)^2 + y^2}}.$$  

So

$$u(x, y) = \frac{2x^2 - 2y^2 + 3}{\sqrt{(x - 1)^2 + y^2}}, \quad v(x, y) = \frac{4xy}{\sqrt{(x - 1)^2 + y^2}}$$

and $u, v$ undefined $\iff (x - 1)^2 = 0 \& y^2 = 0 \iff x = 1 \& y = 0$

Thus $\text{dom}(f) = \mathbb{C}\{1\}$.

(b) $\text{Re}(z) > 5 \Rightarrow \text{Re}(f(z)) > 10$. So $\text{Ran}(f) = \{w \in \mathbb{C} : \text{Re}(w) > 10\}$.

(c) Let $z = re^{i\theta}$. Then

$$f(z) = -2re^{3i\theta} = 2re^{i(3\theta + \pi)}.$$  

So $|f(z)| = 2r$, $\arg(f(z)) = 3\theta + \pi$.

Now enforce the restrictions on the domain:

$$|z| < 1 \Rightarrow |f(z)| < 2$$

and as $\theta = \arg(z)$

$$0 < \theta < \pi/2 \Rightarrow 0 < 3\theta < 3\pi/2 \Rightarrow \pi < 3\theta + \pi < 5\pi/2 \Rightarrow -\pi < \arg(f(z)) < \pi/2.$$  

Thus $\text{Ran}(f) = \{w \in \mathbb{C} : |w| < 2, \ -\pi < \arg(w) < \pi/2\}$

(d) Let $z = re^{i\theta}$, and as $|z| = 1$ then $r = 1$. So

$$f(z) = \frac{1}{2}(e^{i\theta} + e^{-i\theta}) = \cos(\theta).$$

But $\theta \in \mathbb{R}$, so we in fact have $\text{Ran}(f) = [-1, 1] \subseteq \mathbb{R}$. 