1. Answer is on last weeks sheet.

2. (a) 
\[
\lim_{z \to i \sqrt{2}} \frac{2z^2 + 4}{z^4 - 4} = \lim_{z \to i \sqrt{2}} \frac{2(z^2 + 2)}{(z^2 - 2)(z^2 + 2)} = \frac{2}{\frac{\sqrt{2}}{z} - 2} = \frac{2}{(-2) - 2} = -\frac{1}{2}
\]

(b) If \(z = x + iy\), then \(f(z) = e^{x+iy} = e^x e^{iy} = e^x \cos y + ie^x \sin y\).

As \(e^x \cos y\) and \(e^x \sin y\) are products of continuous everywhere functions in \(\mathbb{R}^2\), they are continuous everywhere themselves, hence \(f(z)\) is continuous on all of \(\mathbb{C}\).

(c) Consider the limit inside the exponential first.

\[
\lim_{z \to -i\pi} \frac{z^2 + \pi^2}{z + i\pi} = \lim_{z \to -i\pi} \frac{(z + i\pi)(z - i\pi)}{z + i\pi} = \lim_{z \to -i\pi} z - i\pi = -2\pi i
\]

As \(\exp(u)\) is continuous for all \(u \in \mathbb{C}\),

\[
\lim_{z \to -i\pi} \exp\left(\frac{z^2 + \pi^2}{z + i\pi}\right) = \exp\left(\lim_{z \to -i\pi} \frac{z^2 + \pi^2}{z + i\pi}\right) = \exp(-2\pi i) = 1
\]

(d) 
\[
\lim_{z \to -i\pi/6} \cosh z = \lim_{z \to -i\pi/6} \frac{1}{2}(e^z + e^{-z}) = \frac{1}{2}(e^{i\pi/6} + e^{-i\pi/6}) = \cos(\pi/6) = \frac{\sqrt{3}}{2}
\]

(e) 
\[
\lim_{z \to -i\pi/4} \sin z = \lim_{z \to -i\pi/4} \frac{1}{2i}(e^{iz} - e^{-iz}) = \frac{i}{2}(e^{-\pi/4} - e^{\pi/4}) = \frac{i}{2}(e^{\pi/4} - e^{-\pi/4}) = i \sinh\left(\frac{\pi}{4}\right)
\]

(f) Now \(e^{\pi z/2} - i \to e^{i\pi/2} - i = i - i = 0\) as \(z \to i\), and \(z - i \to 0\) as \(z \to i\), so

\[
\lim_{z \to i} \frac{e^{\pi z/2} - i}{z - i} = \lim_{z \to i} \frac{\pi e^{i\pi/2}}{1} \quad \text{by L'Hopitals rule}
\]

\[
= \frac{\pi e^{i\pi/2}}{2}
\]

\[
= \frac{\pi}{2}
\]
3. 
\[
\lim_{{z \to \infty}} (1 + z^{-2}) = \lim_{{z \to 0}} \left( 1 + \left( \frac{1}{z} \right)^{-2} \right) = \lim_{{z \to 0}} (1 + z^2) = 1
\]

4. We have \( u(x, y) = x, \ v(x, y) = y^2 \), so
\[
\frac{\partial u}{\partial x} = 1, \quad \frac{\partial v}{\partial x} = 0, \quad \frac{\partial u}{\partial y} = 0, \quad \frac{\partial v}{\partial y} = 2y.
\]
All partial derivatives are continuous, so \( u, v \) are \( C^1 \). For the Cauchy-Riemann equations to hold, we need
\[
\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}, \quad \text{which occurs everywhere}
\]
and
\[
\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \Leftrightarrow 2y = 1 \Leftrightarrow y = \frac{1}{2}
\]
So \( f \) is differentiable on the set \( \{ z \in \mathbb{C} : \Im(z) = 1/2 \} \).

5. (a) We saw above that \( f \) was differentiable only on a line, so it is not differentiable on any open set. Thus \( f \) is nowhere analytic.

(b) Let \( z = x + iy \), then
\[
w(z) = z^2 = (x^2 - y^2) + i(2xy), \quad \text{and } u(x, y) = x^2 - y^2, \ v(x, y) = 2xy.
\]
From this we see that
\[
\frac{\partial u}{\partial x} = 2x = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -2y, \quad \frac{\partial v}{\partial x} = 2y.
\]
These partial derivatives are all continuous, so \( u, v \) are \( C^1 \) on \( \mathbb{C} \), and the Cauchy-Riemann equations hold for all \( z \in \mathbb{C} \). Thus \( w \) is entire.

(c) I am not providing a written solution to this question, as I anticipate few people will get up to it in class. It is a slightly tricky one, please attempt it and let me know if you have any trouble.