1. In an experiment on washing machines, 12 machines are used, 2 of each type (standard and deluxe) from each of 3 companies. Four washings are made with each machine. The data are shown below.

(a) Which (pairs of) factors are crossed and which are nested?
(b) Which of the factors should, most likely, be treated as random?
(c) Analyze the data to determine which factors are significant and give estimates of the variances of the random component(s).

<table>
<thead>
<tr>
<th>Company</th>
<th>A Machine</th>
<th>B Machine</th>
<th>C Machine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>1 2</td>
<td>3 4</td>
<td>5 6</td>
</tr>
<tr>
<td></td>
<td>9 7</td>
<td>12 11</td>
<td>28 24</td>
</tr>
<tr>
<td></td>
<td>11 8</td>
<td>9 8</td>
<td>24 23</td>
</tr>
<tr>
<td></td>
<td>12 10</td>
<td>11 11</td>
<td>25 24</td>
</tr>
<tr>
<td></td>
<td>8 6</td>
<td>14 9</td>
<td>26 25</td>
</tr>
<tr>
<td>Deluxe</td>
<td>7 8</td>
<td>9 10</td>
<td>11 12</td>
</tr>
<tr>
<td></td>
<td>16 18</td>
<td>13 19</td>
<td>33 34</td>
</tr>
<tr>
<td></td>
<td>14 17</td>
<td>14 17</td>
<td>31 32</td>
</tr>
<tr>
<td></td>
<td>13 19</td>
<td>12 12</td>
<td>29 33</td>
</tr>
<tr>
<td></td>
<td>15 14</td>
<td>15 18</td>
<td>30 32</td>
</tr>
</tbody>
</table>

2. The data below were obtained from a study of the effect of oven temperature and baking time on the life (in hours) of an electrical component.

<table>
<thead>
<tr>
<th>Oven Temperature (°F)</th>
<th>Baking Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td>600</td>
<td>165</td>
</tr>
<tr>
<td></td>
<td>220</td>
</tr>
<tr>
<td></td>
<td>212</td>
</tr>
<tr>
<td>620</td>
<td>170</td>
</tr>
<tr>
<td></td>
<td>192</td>
</tr>
<tr>
<td></td>
<td>181</td>
</tr>
<tr>
<td>640</td>
<td>126</td>
</tr>
<tr>
<td></td>
<td>160</td>
</tr>
<tr>
<td></td>
<td>149</td>
</tr>
</tbody>
</table>

The experiment was conducted using a split-plot design. There were nine firings of the oven, three firings at each of the three temperatures, with three components per firing and one component removed after 5, 10 and 15 minutes.

(a) Carry out an analysis appropriate for this design to determine which effects are significant.
(b) Give estimates of the whole-plot and sub-plot error variances.
(c) Find standard errors appropriate for comparing (2) levels of any significant factors.
1 Solutions

1. (a) Company and Type are crossed, Machine is nested within Company × Type.

(b) Probably reasonable to treat machines as random.

(c) R analysis:

```r
> wash <- read.csv("../data/wash.csv")
> require(nlme)
> str(wash)
'data.frame': 48 obs. of 5 variables:
$ response: int 9 11 12 8 7 8 10 6 12 9 ...
$ company : int 1 1 1 1 1 1 1 1 2 2 ...
$ type : int 1 1 1 1 1 1 1 1 1 1 ...
$ mach : int 1 1 1 1 2 2 2 2 3 3 ...
$ mach2 : int 1 1 1 1 2 2 2 2 1 1 ...
> wash$company <- factor(wash$company)
> wash$type <- factor(wash$type)
> wash$mach <- factor(wash$mach)
> wash.1 <- lme(response ~ company * type, random = ~1 | mach, data = wash)
> Figure 1 has diagnostics - these look fine except for the almost bimodal machine random effects.

> par(mfrow = c(3, 2), las = 1, mar = c(4, 4, 3, 2))
> scatter.smooth(fitted(wash.1, level = 0), wash$response,
+ ylab = "Observed", xlab = "Fitted")
> abline(0, 1)
> scatter.smooth(fitted(wash.1), residuals(wash.1), xlab = "Fitted",
+ ylab = "Residuals")
> abline(h = 0)
> scatter.smooth(fitted(wash.1), sqrt(abs(residuals(wash.1))),
+ xlab = "Fitted", ylab = "Sqrt(abs(Residuals))")
> qqnorm(residuals(wash.1), main = "Residuals")
> qqline(residuals(wash.1))
> qqnorm(ranef(wash.1)[[1]], main = "Machines")
> qqline(ranef(wash.1)[[1]])

Move on to inference.

> anova(wash.1)

<table>
<thead>
<tr>
<th></th>
<th>numDF</th>
<th>denDF</th>
<th>F-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>1</td>
<td>36</td>
<td>1501.0822</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>company</td>
<td>2</td>
<td>6</td>
<td>130.4973</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>type</td>
<td>1</td>
<td>6</td>
<td>43.1726</td>
<td>0.0006</td>
</tr>
<tr>
<td>company:type</td>
<td>2</td>
<td>6</td>
<td>0.8214</td>
<td>0.4838</td>
</tr>
</tbody>
</table>

Omit the non-significant company:type interaction.

> wash.2 <- lme(response ~ company + type, random = ~1 | mach, data = wash)

> Figure 2 has diagnostics - these look fine.

> par(mfrow = c(3, 2), las = 1, mar = c(4, 4, 3, 2))
> scatter.smooth(fitted(wash.2, level = 0), wash$response,
+ ylab = "Observed", xlab = "Fitted")
> abline(0, 1)
> scatter.smooth(fitted(wash.2), residuals(wash.2), xlab = "Fitted",
+ ylab = "Residuals")
> abline(h = 0)
> scatter.smooth(fitted(wash.2), sqrt(abs(residuals(wash.2))),
+ xlab = "Fitted", ylab = "Sqrt(abs(Residuals))")
> qqnorm(residuals(wash.2), main = "Residuals")
> qqline(residuals(wash.2))
> qqnorm(ranef(wash.2)[[1]], main = "Machines")
> qqline(ranef(wash.2)[[1]])

2
Figure 1: Diagnostic plots for base washing machine model
+ ylab = "Residuals")
> abline(h = 0)
> scatter.smooth(fitted(wash.2), sqrt(abs(residuals(wash.2))),
+ xlab = "Fitted", ylab = "Sqrt(abs(Residuals))")
> qqnorm(residuals(wash.2), main = "Residuals")
> qline(residuals(wash.2))
> qqnorm(ranef(wash.2)[[1]], main = "Machines")
> qline(ranef(wash.2)[[1]])

Assess the model.
> anova(wash.2)

<table>
<thead>
<tr>
<th></th>
<th>numDF</th>
<th>denDF</th>
<th>F-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>1</td>
<td>36</td>
<td>1571.2520</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>company</td>
<td>2</td>
<td>8</td>
<td>136.5975</td>
<td>.0001</td>
</tr>
<tr>
<td>type</td>
<td>1</td>
<td>8</td>
<td>45.1908</td>
<td>1e-04</td>
</tr>
</tbody>
</table>

Estimate the fixed effects and variance components.
> summary(wash.2)$tTable

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Std.Error</th>
<th>DF</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>9.291667</td>
<td>0.8987356</td>
<td>36</td>
<td>10.3385986</td>
<td>2.532982e-12</td>
</tr>
<tr>
<td>company2</td>
<td>0.500000</td>
<td>1.1007218</td>
<td>8</td>
<td>0.4542474</td>
<td>6.617217e-01</td>
</tr>
<tr>
<td>company3</td>
<td>16.000000</td>
<td>1.1007218</td>
<td>8</td>
<td>14.5359160</td>
<td>4.915389e-07</td>
</tr>
<tr>
<td>type2</td>
<td>6.041667</td>
<td>0.8987356</td>
<td>8</td>
<td>6.7224071</td>
<td>1.492229e-04</td>
</tr>
</tbody>
</table>

> VarCorr(wash.2)

mach = pdLogChol(1)

<table>
<thead>
<tr>
<th></th>
<th>Variance</th>
<th>StdDev</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>1.633247</td>
<td>1.277985</td>
</tr>
<tr>
<td>Residual</td>
<td>3.159722</td>
<td>1.777561</td>
</tr>
</tbody>
</table>

Conclusions:
- Both Company and Type are highly significant and there is no evidence of (significant) interaction between Company and Type.
- The estimate of the within machine variance is 3.160 and the estimate of the between machine variance is 1.633.

Marking Key: 8 possible marks

(a) 1 point

(b) 1 point

(c) 6 points as follows
i. 1 point for producing diagnostics at least once, 1 point for interpretation
ii. 1 point for reasonable fitting strategy (eg stepwise)
iii. 1 point for correct final model
iv. 1 point for reporting fixed effects.
v. 1 point for reporting random effects.
Figure 2: Diagnostic plots for additive washing machine model
2. `ovens <- data.frame(time = factor(rep(c(5, 10, 15), each = 9)),
+            temp = factor(rep(rep(c(600, 620, 640), each = 3),
+                     3)), firing = factor(rep(1:9, 3)), life = c(165,
+                     220, 212, 170, 192, 181, 126, 160, 149, 196,
+                     228, 232, 176, 224, 217, 156, 138, 157, 170,
+                     203, 198, 168, 189, 195, 116, 142, 159))`

`ovens.1 <- lme(life ~ time * temp, random = ~1 | firing,
+            data = ovens)`

Figure 3 has diagnostics - these look fine.

Par(mfrow = c(3, 2), las = 1, mar = c(4, 4, 3, 2))

`scatter.smooth(fitted(ovens.1, level = 0), ovens$life,
+            ylab = "Observed", xlab = "Fitted")

abline(0, 1)

`scatter.smooth(fitted(ovens.1), residuals(ovens.1), xlab = "Fitted",
+            ylab = "Residuals")

abline(h = 0)

`scatter.smooth(fitted(ovens.1), sqrt(abs(residuals(ovens.1))),
+            xlab = "Fitted", ylab = "Sqrt(abs(Residuals))")

qqnorm(residuals(ovens.1), main = "Residuals")

qqline(residuals(ovens.1))

`qqnorm(ranef(ovens.1)[[1]], main = "Machines")

Move on to inference.

`anova(ovens.1)`

<table>
<thead>
<tr>
<th>numDF</th>
<th>denDF</th>
<th>F-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>1</td>
<td>12</td>
<td>968.6754</td>
</tr>
<tr>
<td>time</td>
<td>2</td>
<td>12</td>
<td>8.1448</td>
</tr>
<tr>
<td>temp</td>
<td>2</td>
<td>6</td>
<td>9.3340</td>
</tr>
<tr>
<td>time:temp</td>
<td>4</td>
<td>12</td>
<td>0.8117</td>
</tr>
</tbody>
</table>

Omit interaction.

`ovens.2 <- lme(life ~ time + temp, random = ~1 | firing,
+            data = ovens)`

Figure 4 has diagnostics - these look fine.

Par(mfrow = c(3, 2), las = 1, mar = c(4, 4, 3, 2))

`scatter.smooth(fitted(ovens.2, level = 0), ovens$life,
+            ylab = "Observed", xlab = "Fitted")

abline(0, 1)

`scatter.smooth(fitted(ovens.2), residuals(ovens.2), xlab = "Fitted",
+            ylab = "Residuals")

abline(h = 0)

`scatter.smooth(fitted(ovens.2), sqrt(abs(residuals(ovens.2))),
+            xlab = "Fitted", ylab = "Sqrt(abs(Residuals))")

qqnorm(residuals(ovens.2), main = "Residuals")

qqline(residuals(ovens.2))

`qqnorm(ranef(ovens.2)[[1]], main = "Machines")

`> anova(ovens.2)`

<table>
<thead>
<tr>
<th>numDF</th>
<th>denDF</th>
<th>F-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>1</td>
<td>12</td>
<td>968.6754</td>
</tr>
<tr>
<td>time</td>
<td>2</td>
<td>12</td>
<td>8.1448</td>
</tr>
<tr>
<td>temp</td>
<td>2</td>
<td>6</td>
<td>9.3340</td>
</tr>
<tr>
<td>time:temp</td>
<td>4</td>
<td>12</td>
<td>0.8117</td>
</tr>
</tbody>
</table>
Figure 3: Diagnostic plots for base ovens model
The plots in Figure 4 represent diagnostic plots for the base ovens model. They include scatter plots of observed vs. fitted values, residuals vs. fitted values, square root of absolute residuals vs. fitted values, and theoretical quantiles vs. sample quantiles. These plots are used to check the assumptions of a linear regression model, such as linearity, homoscedasticity, and normality of residuals.
Next, obtain the estimates to answer the questions.

```r
> anova(ovens.2)

            numDF denDF  F-value p-value
(Intercept)    1    16 968.6797  <.0001
  time          2    16  8.5470  0.0030
  temp          2     6  9.3341  0.0144

> VarCorr(ovens.2)

  firing = pdLogChol(1)

    Variance StdDev
  (Intercept) 257.0621 16.03316
  Residual     124.1111 11.14051

> options()$contrasts

 unordered ordered
 "contr.treatment" "contr.poly"

> summary(ovens.2)$tTable

                        Value Std.Error  DF  t-value p-value
(Intercept)          198.444444 10.424533 16 19.0362905 2.046219e-12
  time10            16.555556  5.251689 16  3.1524251 6.163754e-03
  time15            -3.888889  5.251689 16 -0.7405025 4.697233e-01
  temp620          -12.444444 14.105140  6 -0.8822631 4.115828e-01
  temp640         -57.888889 14.105140  6 -4.1040989 6.328154e-03
```

(a) **Conclusions:** The interaction between (Oven) Temperature and (Baking) Time is not significant (P = 0.541); omit it. The effects of Temperature and Time are both significant (P = 0.014 and 0.003, respectively).

(b) The estimate of the sub-plot error variance is 124.11 and the estimate of the whole-plot error variance is 257.1.

(c) The standard error of the estimate of the difference between two levels of Temperature is 14.11, and the standard error of the estimate of the difference between two levels of Time is 5.25.

Marking Key: 7 possible marks

(a) 1 point for producing diagnostics at least once, 1 point for interpretation

(b) 1 point for reasonable fitting strategy (eg stepwise)

(c) 1 point for correct final model

(d) 1 point for reporting fixed effects.

(e) 1 point for reporting random effects.

(f) 1 point for reporting standard errors of differences.