Subject code: 620-157

Subject name: Mathematics 1

Credit points: 12.5

Coordinator: Karen Baker

Semesters of offer: 1

Prerequisites: Study score of 40 or more in VCE Specialist Mathematics 3/4 or equivalent, or permission from the Director of the Mathematics and Statistics Learning Centre

Mode of delivery: Lectures, tutorials and computer laboratory classes.

Contact hours: 48 one-hour lectures (four per week), 11 one-hour tutorials (one per week), 11 one-hour computer laboratory classes (one per week)

Estimated total time commitment: 120 hours

Description: This subject develops the concepts of vectors, matrices, sequences and the methods of linear algebra. Students should gain an appreciation of mathematical proof. Little of the material here has been seen at school and the level of understanding required represents an advance on previous studies. Underlying concepts developed in lectures will be reinforced in computer laboratory classes. Systems of linear equations, matrices and determinants, vector geometry, lines and planes, vector spaces, subspaces, linear independence, bases, dimension, inner products, linear transformations, eigenvalues, eigenvectors. Foundations of analysis, techniques of proof and heuristic and rigorous discussion of convergence of sequences. Complex numbers.

Assessment: Up to 25 pages of written assignments 10% (due during semester), two 45-minute written computer laboratory tests 10% (held during semester), a 3-hour written examination 80% (in the examination period).

Prescribed texts: H. Anton and C. Rorres, *Elementary Linear Algebra, Applications Version*, 9th edition, Wiley, 2005.

Notes: This is the first subject of a three-subject sequence (620-157, 620-158 and 620-2xx Multivariable and Vector Calculus) for students with a very high level of achievement in VCE Specialist Mathematics or equivalent. This subject sequence is equivalent, in content, to the four subjects 620-154, 620-156, 620-2xx Vector Calculus and 620-2xx Real Analysis with Applications, presenting some topics from a more advanced perspective.
Students may only gain credit for one of [07]620-122, [08]620-142, 620-156, 620-157, [05]620-192, [05]620-194 or [07]610-211.
Students in the combined degree BE/BSc should note that credit exclusions exist between this subject and Engineering mathematics subjects. Refer to entries for 431-201 Engineering Analysis A and 431-202 Engineering Analysis B for details.

Subject objectives: Students completing this subject will:

- use matrix techniques to represent and solve a system of simultaneous linear equations;
- understand the extension of vector concepts to abstract vector spaces of arbitrary finite dimension;
- understand linear transformations, their matrix representations and applications;
- understand the concept and properties of sequences;
- perform calculations with complex numbers;
- become familiar with the use of a computer package for symbolic and numeric calculation.

Generic skills: In addition to learning specific skills that will assist students in their future careers in science, they will have the opportunity to develop generic skills that will assist them in any future career path. These include

- problem-solving skills: the ability to engage with unfamiliar problems and identify relevant solution strategies;
- analytical skills: the ability to construct and express logical arguments and to work in abstract or general terms to increase the clarity and efficiency of analysis;
- collaborative skills: the ability to work in a team;
- time management skills: the ability to meet regular deadlines while balancing competing commitments.
- computer skills: the ability to use an appropriate computing package.

Lecture-by-lecture outline:

Linear Equations

1. Systems of linear equations, geometrical interpretation, augmented matrices, row operations.

2. Gaussian elimination to row echelon form and reduced row echelon form.

3. Rank, consistent and inconsistent systems.

Matrices

4. Matrices, matrix algebra, special matrices.

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5. Matrices for elementary row operations.

6. Matrix inverses, augmented matrices, matrix equations.

Linear Independence / Determinants

7. Linear independence, solution spaces, row and column spaces.

8. Determinants, cofactors.

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9. Determinants by row operations, properties.

Number Systems and Methods of Proof

10. Number systems: $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ (constructed from $\mathbb{Z}$).

11. Proof by contradiction, infinitely many primes, $\sqrt{2}$ not rational.

12. Mathematical induction; simple examples involving proving identities.

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13. Mathematical induction; more challenging examples including proving inequalities.

Vectors / Solid Geometry

14. Abstract vectors; $\mathbb{R}^n$, geometrical vectors, Norm in $\mathbb{R}^n$, dot product in $\mathbb{R}^n$.

15. Cross and triple products in $\mathbb{R}^3$.

16. Solid geometry, equations of lines and planes.

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17. Applications of lines and planes.

Vector Spaces

18. Real vector spaces, examples other than $\mathbb{R}^n$.

19. Subspaces.

20. Linear combinations, spanning sets.

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21. Linear dependence and independence.

22. Bases and dimension, coordinates.

23. Inner products on vector spaces.

24. GOOD FRIDAY/EASTER BREAK (nominal position).

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25. Norm, orthogonality, angles, Cauchy-Schwartz inequality.

26. Orthonormal bases, Gram-Schmidt process.

27. Least squares estimation, curve fitting.

28. MID-SEMESTER TEST (nominal position).

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29. Linear transformations, matrix representations.

30. Linear transformations, matrix representations (continued).

31. Image and kernel spaces, rank and nullity.

32. ANZAC DAY (nominal position)

Eigenvalues, Eigenvectors

33. Eigenvalues and eigenvectors, characteristic polynomial.

34. Similarity transformations and diagonalization.

35. Matrix powers, Cayley-Hamilton theorem (not proved generally).

36. Real symmetric and orthogonal matrices.

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37. Miscellaneous examples on eigenvalues and eigenvectors.

Sequences

38. Sequences and convergence, examples and heuristics.

39. ϵ-N definition of convergence.

40. Useful inequality arguments.

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41. Further examples on ϵ-N arguments.

42. Limit theorems, sandwich theorem, standard limits.

43. Supremum, infimum, monotone sequence theorem, completeness.

44. Applications of sequences.

Complex numbers

45. Complex numbers, polar form, de Moivre's theorem proven by induction, complex exponential

46. Fractional powers, construction of $\mathbb{C}$ from $\mathbb{R}$.

47. Complex vector spaces.

48. Revision.
