Subject code: 620-158

Subject name: Mathematics 2

Credit points: 12.5

Coordinator: Karen Baker

Semesters of offer: 2

Prerequisites: 620-157 Mathematics I

Mode of delivery: Lectures and tutorials.

Contact hours: 48 one-hour lectures (four per week), 11 one-hour tutorials (one per week)

Estimated total time commitment: 120 hours

Description: This subject develops fundamental concepts and principles in mathematical analysis. Students should gain skills in the practical techniques of differential calculus, integral calculus and infinite series, and study selected applications of these techniques in mathematical modelling. Heuristic and rigorous discussion of limits of real-valued functions, continuity and differentiability. Mean Value Theorem and applications, Taylor polynomials. Riemann integration, techniques of integration and applications, improper integrals. Infinite series. First order differential equations, second order linear differential equations with constant coefficients and selected applications.

Assessment: Up to 25 pages of written assignments 10% (due during semester), a 45-minute written test 10% (held mid-semester), a 3-hour written examination 80% (in the examination period).


Notes: This is the second subject of a three-subject sequence (620-157 Mathematics 1, 620-158 Mathematics 2 and 620-2xx Multivariable and Vector Calculus) for students with a very high level of achievement in VCE Specialist Mathematics or equivalent. This subject sequence is equivalent, in content, to the four subjects 620-154, 620-156, 620-2xx Vector Calculus and 620-2xx Real Analysis with Applications, presenting some topics from a more advanced perspective. Students may only gain credit for one of [07]620-113, [07]620-123, [08]620-143, 620-155, 620-158 or [05]620-193.

Subject objectives: Students completing this subject will:

- understand the significance and applications of properties of functions such as limits, continuity and differentiability;
- be exposed to some fundamental results in real analysis such as the Mean Value Theorem;
- be able to evaluate proper and improper Riemann integrals;
• develop the ability to determine the convergence and divergence of infinite series;

• solve analytically first and second order ordinary differential equations, and use these equations to model some simple physical systems.

Generic skills: In addition to learning specific skills that will assist students in their future careers in science, they will have the opportunity to develop generic skills that will assist them in any future career path. These include

• problem-solving skills: the ability to engage with unfamiliar problems and identify relevant solution strategies;

• analytical skills: the ability to construct and express logical arguments and to work in abstract or general terms to increase the clarity and efficiency of analysis;

• collaborative skills: the ability to work in a team;

• time management skills: the ability to meet regular deadlines while balancing competing commitments.

Lecture-by-lecture outline:

Limits

1. Functions in the general sense. Injections, bijections, surjections.

2. Limits of real-valued functions at a point and at infinity: heuristics and $\varepsilon-\delta$ definition.

3. Establishing the existence of limits from the definition. Relevant inequality manipulations.

4. Algebra of limits, limit theorems, Sandwich Theorem.

Continuity

5. Continuity at a point ($\varepsilon-\delta$). Sum, product, quotient, composition of continuous functions.

6. Limit of the sequence $f(a_n)$ when $a_n \to a$ and $f$ is continuous at $a$. Euler’s limit of $(1 + x/n)^n$.

7. Continuity on open intervals. One-sided limits, continuity on closed intervals.

8. Intermediate Value Theorem and applications (e.g., root-finding by bisection).

Differential Calculus

9. Differentiability at a point and on open and closed intervals. Differentiation rules.

10. Rolle’s Theorem, Mean Value Theorem. Applications to theorems in elementary calculus.

12. Solution of the nonlinear equation \( x = f(x) \) by iteration: fixed points, stability.

Taylor Expansions

15. \( n \)-th-order Mean Value Theorem. Taylor polynomials, expansion with Lagrange remainder.

Riemann Integration

17. Riemann sums, Riemann integral, integrability of continuous functions with bounded derivative.

Integration and Applications

22. Inverse trigonometric and inverse hyperbolic functions.

Improper Integrals

26. Improper Riemann integrals: finite intervals, unbounded integrands.
27. Improper Riemann integrals: infinite intervals.
28. Convergence tests for improper integrals.

Infinite Series

30. Partial sums, convergence of infinite series, geometric, telescoping and harmonic series.
31. Series of positive terms: comparison and integral tests.
32. Convergent series may be added. Absolute vs conditional convergence. Alternating series.

33. Examples.
35. Power series for the arctan function and the natural logarithm.
36. Applications of power series.

First-order ordinary differential equations
37. First-order ODEs: direction fields, solution curves, existence and uniqueness of solutions.
38. First-order linear ODEs, integrating factors, applications.
40. Applications of first-order ordinary differential equations.

41. Applications of first-order ordinary differential equations (contd).

Second-order linear ordinary differential equations
42. Homogeneous problems. Linearly independent solutions, Wronskian, persistence of independence. Solutions as a vector space.
43. General problem: complementary function, particular integral, reduction of order.
44. Constant coefficient case: applications of complex exponential.

45. Constant coefficient case: finding particular integrals, including superposition ideas.
46. Applications of second order differential equations.
47. Applications of second order differential equations.
48. Constant coefficient case: mechanical oscillations