Honours Project Seminars

Richard Berry Building
Friday 24 November 2006

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Lunch will be served in the staff tea room after the seminars.
Candice Rabusa  
**Supervisor:** David Coulson  
**Project Title:** The Local Kronecker Weber theorem  
**ABSTRACT:**  
The Local Kronecker-Weber theorem states that any finite abelian extension $K$ of the rational $p$-adic field, $\mathbb{Q}_p$, must be contained in $\mathbb{Q}_p(\zeta_n)$ for some $n \in \mathbb{N}$. Since every abelian field is a compositum of fields of prime power degree, the Local K-W can be shown by considering $\text{Gal}(K/\mathbb{Q}_p) \cong (\mathbb{Z}/q^m\mathbb{Z})$, which is cyclic of degree $q^m$ for any prime $q$ and $m \geq 1$. This can be broken down into 3 cases: (i) $q \neq p$, (ii) $q = p \neq 2$, and (iii) $q = p = 2$. In this talk, we will outline how the three cases fit together to form our proof. Cases (ii) and (iii) are dealt with by finding the compositum of their respective unramified ($K_u$) and totally ramified extensions ($K_r$). In Case (ii), we obtain $K_u = \mathbb{Q}_p(\zeta_{p^m - 1}, \zeta_{p^m + 1})$ and $K_r = \mathbb{Q}_p(\zeta_{p^m + 1})$ so that $K_u K_r = \mathbb{Q}_p(\zeta_{p^m - 1}, \zeta_{p^m + 1})$. If $K \not\subseteq \mathbb{Q}_p(\zeta_{p^m - 1}, \zeta_{p^m + 1})$, then we can find a subfield $N$ of $K(\zeta_{p^m - 1}, \zeta_{p^m + 1})$ such that $\text{Gal}(N/\mathbb{Q}_p) \cong (\mathbb{Z}/p\mathbb{Z})^3$, which forms a contradiction. Case (iii) follows the same procedure with $N$ having $\text{Gal}(N/\mathbb{Q}_2) \cong (\mathbb{Z}/2\mathbb{Z})^4$ or $\cong (\mathbb{Z}/4\mathbb{Z})^3$. Lastly, case (i) gives a totally and tamely ramified subextension $K/L$ over $\mathbb{Q}_p$, which is inside the field $\mathbb{Q}_p(\zeta_{mnp})$ for $\gcd(m, n, p) = 1$.

Nicholas Sheridan  
**Supervisor:** Craig Hodgson  
**Project Title:** Hamilton’s Ricci Flow  
**ABSTRACT:**  
The Ricci Flow, introduced by Richard Hamilton in 1982, is a partial differential equation that can be applied to the metric on a Riemannian manifold to make the manifold ‘rounder’. It has recently (2003) been used to prove the Geometrization Conjecture, from which the Poincaré Conjecture (one of the biggest unsolved problems of 20th century mathematics) follows. We will introduce some of the basic geometric ideas needed to define the Ricci flow, then some of the techniques used to understand its behaviour. Our aim will be to give an outline of the way Hamilton, in his original 1982 paper, used these techniques to prove a result that relates the Ricci curvature of a manifold (a geometric property) to its topology.

Daniel Yeow  
**Supervisor:** Lawrence Reeves  
**Project Title:** An Introduction to Thompson’s Group  
**ABSTRACT:**  
Many groups have interesting properties which we may study to gain a deeper understanding of mathematics. Thompson’s group presents us with a group that has many beautiful properties and many different, yet logically equivalent ways of looking at them. From the open-ended question of amenability to its direct practical application in cryptography, Thompson’s group, originally the fruit of the work of a logician, gives us a concrete framework in which to study some very abstract concepts.
Peng Wang  
**Supervisor:** Ken Sharpe  
**Project Title:** Exponential Random Graph (p*) Models for Affiliation Networks.  
**ABSTRACT:**  
Statistical modeling of social networks as complex systems has always been and remains a challenge for social scientists. Exponential family models give us a convenient way of expressing local network structures that have sufficient statistics for their corresponding parameters. This kind of model, known as Exponential Random Graph Models (ERGMs), or p* models, have been developed since the 1980s. However, due to the difficulty of dealing with the intractable normalizing constant, pseudo-likelihood estimation methods have been applied in most studies. Recently, simulation based MCMC maximum likelihood estimation techniques have been developed. Furthermore, current advances in the ERGM provides a much better chance of model convergence for large networks compared with the traditional Markov models. To date most work on ERGMs has focused on one-mode networks, and little has been done on applying maximum likelihood estimation in the case of affiliation networks with two or more modes. This paper considers the application of MCMC maximum likelihood estimation to affiliation networks. Similar techniques have been applied to affiliation networks as in the latest specification for one-mode networks. We investigated features of the model by simulation, and compared the goodness of fit results obtained using the maximum likelihood and pseudolikelihood approaches. Examples used in this paper show that the ERGM with the newly specified statistics is a powerful tool for statistical analysis of affiliation networks.

Fanny Sampurno  
**Supervisor:** Ken Sharpe  
**Project Title:** Identifying risk factors associated with new onset cardiovascular disease in patients with type I diabetes using Classification Tree  
**ABSTRACT:**  
This study examines type I diabetes patients without a history of cardiovascular complications, to define which risk factors are most important in the pathogenesis of cardiovascular disease (CVD) in diabetes. Recursive Partitioning and Regression Trees (RPART) statistical software was used for data analysis. This presentation will focus on Classification Tree models which were used to investigate which risk factors predict CVD events during nine year follow-up in 136 type I diabetic patients aged 20-62 years.
ABSTRACT:
In agricultural research, the critical period for weed control (CPWC) is defined as the time during crop development when competition with weeds will adversely affect growth, leading to a decrease in final yield. The CPWC is different for different crops, and may also depend on a range of other factors such as soil type, availability of water and nutrients, and latitude or altitude. In this talk I will review the design of experiments into the CPWC, and the statistical methods currently used to analyse resulting data. I will then highlight some problems with published approaches to data analysis, suggest some solutions, and present simulation results to illustrate the advantages and disadvantages of different methods. I will conclude with some criticisms of the experimental design, and some suggestions for further research.

Name: Warren Volk-Makarewicz  
Supervisor: A/Prof Felisa Vázquez-Abad  
Title: Reduced Variance Sensitivity Estimators for Gaussian Systems 
ABSTRACT:
Suppose we have a system of Gaussian random variables $X_i \sim \mathcal{N}(\mu_i, \sigma_{ij})$, $i \in \{1, \ldots, N\}$ in $(\Omega, \mathcal{F}, \mathbb{P})$. We can determine the derivative estimator $D(X; \cdot)$ of a performance estimator $L(X; \cdot)$ which models a property of the system by the expression
\[
\frac{\partial}{\partial \sigma} \mathbb{E}_{\mathbb{P}}[L(X; \sigma)] = \mathbb{E}_{\mathbb{Q}}[D(X; \sigma)]
\] (1)
where $\mathbb{P}$ may equal $\mathbb{Q}$.

This presentation provides an investigation into derivative estimation of Gaussian random variables. We introduce three unbiased derivative estimators; Infinite Perturbation Analysis (IPA), Score Function (SF), and Measure Valued Differentiation (MVD). Theory of derivative estimation is discussed, and for univariate Gaussian random variables we obtain a new result that for zero mean monomial and general $\mathcal{N}(\mu, \sigma^2)$ exponential univariate performance functions, that the MVD estimator coupled with a simulation scheme from Heidergott et. al. (submitted)$^1$ yields the least variance. This is contrary to general wisdom that IPA is the derivative estimator with smallest variance.

We then provide two applications of derivative estimation to Gaussian systems. Firstly, for a vector of random variables, a Stochastic Activity Network is analyzed. Using each derivative estimation scheme, the sensitivity of the first two moments of the completion times are analyzed w.r.t a common standard deviation of the activity arcs in the network.

The second application is an analysis of a stochastic process. In the Black-Scholes model, the author derives a MVD estimator to determine Vega, the sensitivity of the price of a financial option w.r.t the spot volatility of the model. We then compare the result of this estimator to the analytical expression of Vega for European Vanilla, and Lookback call option contracts.

Hannah Fitzgerald  
**Supervisor:** Paul Pearce  
**Project Title:** Physical Combinatorics of Non-Unitary Minimal Models  
**ABSTRACT:**  
The Forrester-Baxter two-dimensional (FB) lattice model is a square lattice with heights of positive integer \(l\), with \(l < p - 1\), where \(p\) is a parameter characterising the model. The Conformal Field Theories (CFTs) that correspond to these models are the minimal models, \(\mathcal{M}(p, p)\), and each theory has a conformal character.  
Studying the FB lattice model, this thesis looks at the one-dimensional configurational sums, which are obtained from the path diagrams of length \(N\) of the local heights. In the limit as \(N \to \infty\), the one-dimensional configurational sums coincide with the conformal characters of the corresponding minimal models.  
Inexplicably, another form of the finitised conformal character appears as the generating function of the eigenenergies of the transfer matrices of the FB models. This form also contains information on the particle content of the theories. This thesis obtains this form of the character from the physical patterns of the zeros for the eigenvalues of the fused transfer matrices for several specific examples, and studies the relation between the zeros and the path diagrams of the local heights for these examples. This leads us to a combinatorial interpretation of the particle content of the theories considered, an example of physical combinatorics.

David Glover  
**Supervisor:** Peter Forrester  
**Project Title:** Equivalence of Construction Methods for Sturmian Words  
**ABSTRACT:**  
Consider an integer grid in the xy-plane. Draw in the line \(y = \alpha x + \rho\), and record a 1 when the line crosses a horizontal grid line, and a 0 when it crosses a vertical grid line. The resulting sequence is called a cutting sequence, and its mathematical properties relating to equivalent characterizations is a main theme. In particular, notions such as two-distance property, infinite chains of reductions and complexity can all be used to give alternative ways to characterise cutting sequences.  
A formula for cutting sequences involving the arithmetic floor and ceiling function is presented. This allows transformation properties with respect to certain substutitions to be studied. From this a question posed by de Bruijn in his prelude to studying Penrose tilings can be answered.
Anthony Mays  
**Supervisor:** Peter Forrester  
**Project Title:** Combinatorial aspects of juggling  
**ABSTRACT:**  
This paper examines the relationship between the practice of juggling and mathematics. First an introduction and brief history of juggling is presented, where we highlight some of the notable events and personalities in juggling and mathematical juggling. In **Section 2** we discuss some of the ways that mathematical ideas have been used to solve juggling problems. The main contribution in this field was the development of siteswap, a compact and powerful notation for describing juggling patterns, that readily lends itself to generalisation. We prove some simple theorems involving this notation. Some other systems of notation are introduced and we discuss some juggling theorems of Claude Shannon. **Section 3** looks at how juggling concepts have been used to prove theorems that are not strictly juggling-related. We develop a theorem for counting the number of periodic juggling patterns with a fixed number of balls, and use this result to prove an identity relating the number of drops of a permutation of a set to the chromatic polynomial of a graph of the set. We also use this result to develop juggling concepts in the context of $q$-binomial numbers, by introducing *juggling cards* which we can use to create arbitrary juggling patterns. This development allows us to easily calculate the Poincaré series of the affine Weyl group $\tilde{A}_{d-1}$. We also find that we can apply these ideas to prove a theorem involving $q$ - Stirling numbers. Vector compositions can also be naturally described by juggling patterns and we prove some identities involving unitary vector compositions before broadening the discussion to include more general vector compositions.

James Plunkett  
**Supervisor:** Dr. Omar Foda  
**Project Title:** An introduction to the algebraic Bethe ansatz  
**ABSTRACT:**  
This talk will be an introduction to basic aspects of one of the major approaches to solving statistical mechanical models, namely the algebraic Bethe ansatz (ABA). Firstly, we consider the (homogeneous) six-vertex model on a finite lattice with periodic boundary conditions and show how the ABA can be used to characterize the eigenvectors and eigenvalues. Next, we consider the (inhomogeneous) six-vertex model with domain wall boundary conditions, and use the ABA to explain how boundary 1-point functions are calculated in determinant form.

Lauren Kate Trumble  
**Supervisor:** Peter Forrester  
**Project Title:** Relating Random Matrix Theory to Queueing Theory  
**ABSTRACT:**  
This thesis investigates the relationships between probability distributions from two different areas of mathematics; random matrix theory and queueing processes. Random matrix theory is used in physics to accurately predict statistical properties of physical systems. It is the eigenvalues of specific matrices that possess this information. Here we generate probability density functions for the eigenvalues from random Hermitian matrices and random positive definite matrices whose entries are complex Gaussian and consider the Gaussian Unitary Ensemble and the Laguerre Unitary Ensemble. The $N \to \infty$ limit of both of these eigenvalue distributions have an exact from that is given in terms of a solu-
tion of a special non-linear equation called the Painlevé II equation. Queueing theory can be applied to all sorts of queues such as people waiting in a queue at a supermarket. We investigate a specific model for queueing processes and describe the different ways such a queue can be represented. The quantity of interest relating to such a queueing process is the exit time of the final job from the final queue. We investigate when the service times are random variables chosen from the exponential distribution and calculate a probability distribution for this final exit time. This formula is precisely that as for the distribution of the largest eigenvalue in the Laguerre Unitary Ensemble of random complex positive definite matrices.

CONTINUUM MODELLING AND OPERATIONS RESEARCH
Friday 24 November 2005, J H Michell Theatre, Chair: A/Prof John Sader

Nicholas MacKenzie
Supervisor: Assoc. Prof. John E. Sader, Co-Supervisor: Dr. Charles R. Lilley
Project Title: Stability of the Beagle 2 Mars Explorer

ABSTRACT:
The European Space Agency’s Mars explorer, the Beagle 2, was lost during entry into the outer atmosphere of the planet. There is evidence to suggest that its disappearance could have been caused by the spherically blunted, conical shape of the landing pod. While in continuous fluid flows this is generally a stabilising shape, it has been proposed that in the world of rarefied (low-density) gases, it is actually unstable and may have contributed to the failure of the Beagle mission. The Direct Simulation Monte Carlo (DSMC) numerical method, as well as analytical modelling, was used to investigate this theory.

The results of both the analytical and numerical study suggest that the stability (or otherwise) of the Beagle as it entered the Martian atmosphere is highly dependent on the position of the centre of mass and the transverse momentum accommodation coefficient (TMAC), particularly for high Knudsen numbers where the gas is rarefied. For TMAC values close to unity or for centres of mass close to the back of the craft, instabilities tended to arise.

Michelle Willcox
Supervisor: John Sader
Project Title: Oscillations of a Thin Blade in Fluid

ABSTRACT:
It is common practice in mathematical publications to use or expand on the work of others. Propagation of erroneous calculations is undesirable, and it is therefore useful to investigate the validity of published work. This talk focuses on a paper published in 1981, results of which have not been rigorously proved in the literature until now. This paper was written by Payne and, while investigating flat plate planing theory, he made assertions regarding forces acting upon rectangular plates. Payne claimed that forces behave like the square-root of the aspect ratio as rectangular plates become infinitely long. These claims are examined in detail to determine the true nature of forces on rectangular plates. Elliptic coordinates, conformal mapping and, in particular, the Schwarz-Christoffel transformation are used along with the aid of asymptotic analysis.
Clare Kitching
Supervisor: Peter Taylor
Project Title: Algorithms for return probabilities for stochastic fluid flows

ABSTRACT:
Stochastic fluid queues are used in modelling buffers in telecommunications systems. The probability that the buffer level will return to its initial level is an indicator of stability and also a useful quantity for calculating other characteristics of the fluid queue. There are several algorithms available to calculate the return probabilities, each of which is suited to a particular type of process. The key to the performance of the algorithms is the physical interpretation, which determines its compatibility with certain processes. We look at a range of these algorithms and consider the physical interpretation and performance in terms of rates of convergence and complexity. Of particular note is the structure-preserving doubling algorithm which has not been applied in this area previously, and we give a physical interpretation of this algorithm in the fluid flow environment.

David Bannister
Supervisor: Natasha Boland, Co-Supervisor: Heng-Soon Gan
Project Title: Analysis of the Military Lift Problem

ABSTRACT:
A network flow problem with multiple modes and commodities can be used to model a range of real-world problems from package delivery to disaster relief operations. This talk examines modelling the problem of troop and equipment transportation, as devised by the Defence Science and Technology Organisation, as well as the solutions to the model. As the delay in arrival of some commodities may result in loss of infrastructure or human life, the problem is modelled with an emphasis on minimising the total time to move all commodities. The formulation will be explained, and an exposition into the use of redundant constraints to improve solving times will be given. In addition, proposed future directions of the model will be considered.

Thomas Knight Gregson
Supervisor: Natasha Boland, Co-Supervisor: Heng-Soon Gan
Project Title: Personnel Scheduling for a Global Tour Company

ABSTRACT:
The Tour Leader Scheduling Problem (TSLP) is introduced as a new combinatorial problem. The motivation for this research is to optimize the human resource planning operations for a company that operates holiday tours around the world. The problem is to create a schedule for the season. A schedule is a complete list of each trip offered by the client with an assigned leader to guide that trip. It is essentially a network flow combinatorial problem; closely related to existing problems such as Home Health Care, Vehicle Routing, Set Covering, Set Packing and others. The optimization is to determine a schedule at minimum cost subject to hard and soft constraints, such as work load, block time and qualifications. An integer programming model is created which encompasses all elements of the problem, however is too large to solve. The model is then decomposed using Dantzig-Wolfe decomposition, and solved using heuristic methods. An analysis of the effectiveness of the heuristic methods is found by using column generation, and comparing small time horizons of the integer programming model. The outcomes show that the heuristic methods provide a good approximation to the problem. An optimal schedule used by the company is also produced.