Subject code: 620-155

Subject name: Calculus 2

Credit points: 12.5

Coordinator: Karen Baker

Semesters of offer: 1, repeat 2

Prerequisites: Study score of 27 or more in VCE Specialist Mathematics 3/4 or equivalent, or one of [07]620-151, 620-154, [07]620-161 or permission from the Director of the Mathematics and Statistics Learning Centre

Mode of delivery: Lectures and tutorials

Contact hours: 36 one-hour lectures (three per week), 12 one-hour tutorials (one per week)

Estimated total time commitment: 120 hours

Description: This subject will extend knowledge of calculus from school. Students are introduced to hyperbolic functions and their inverses, the complex exponential and functions of two variables. Techniques of differentiation and integration will be extended to these cases. Students will be exposed to a wider class of differential equation models, both first and second order, to describe systems such as population models, electrical circuits and mechanical oscillators.
Calculus: intuitive idea of limits, continuity and differentiability of functions of one variable, hyperbolic functions and their inverses, implicit differentiation, level curves, partial derivatives, chain rules for partial derivatives, directional derivative, tangent planes and extrema for functions of several variables. Complex exponential: definition, derivative, integral and applications. Integration: techniques of integration, arc length and double integrals. Ordinary differential equations: first order (separable, linear via integrating factor) and applications, second order constant coefficient (particular solutions, complementary functions) and applications.

Assessment: Up to 25 pages of written assignments 10% (due during semester), a 45-minute written test 10% (held mid-semester), a 3-hour written examination 80% (in the examination period).

Prescribed texts: H. Anton, I. Bivens, S. Davis, *Calculus: Early Transcendentals*, 8th edition, Wiley, 2005.

Notes: Students with a score of 40 or more in Specialist Mathematics 3/4 will normally not be permitted to enrol in this subject; such students should enrol in 620-157.
Students may only gain credit for one of [07]620-113, [07]620-121, [[07]620-123, [07]620-143, 620-155, 620-158.

Subject objectives: Students completing this subject will:

- Understand the intuitive idea of limits, continuity and differentiability of a function of one variable;
- Develop the ability to sketch and manipulate hyperbolic and inverse hyperbolic functions;
- Use differential calculus to solve extrema problems involving functions of several variables.
- Be able to evaluate integrals using trigonometric and hyperbolic substitutions, partial fractions, integration by parts and the complex exponential;
- Solve analytically first and second order ordinary differential equations, and use these equations to model some simple physical systems.

Generic skills: In addition to learning specific skills that will assist students in their future careers in science, they will have the opportunity to develop generic skills that will assist them in any future career path. These include

- problem-solving skills: the ability to engage with unfamiliar problems and identify relevant solution strategies;
- analytical skills: the ability to construct and express logical arguments and to work in abstract or general terms to increase the clarity and efficiency of analysis;
- collaborative skills: the ability to work in a team;
- time management skills: the ability to meet regular deadlines while balancing competing commitments.

Lecture-by-lecture outline:

Functions of One Variable

1. Intuitive idea of limits. Limit laws. Limits involving infinity.

2. Evaluating limits. Sandwich theorem. Continuity of functions.

3. Limits of compositions of functions. Differentiability of functions.

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4. L'Hôpital's rule. Implicit differentiation.

5. Hyperbolic functions: definition, basic properties, sketching graphs.

6. Hyperbolic functions: identities and applications, differentiation.

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7. Inverse trigonometric functions: definition, sketching, differentiation, manipulation.

8. Inverse hyperbolic functions: definition, sketching, differentiation, manipulation.

Complex Numbers

9. Complex exponential: definitions and properties.

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10. Complex exponential: applications and its derivative.

Integral Calculus

11. Trigonometric and hyperbolic substitutions. Products of hyperbolic functions.

12. Partial fractions. Polynomial long division before decomposing into partial fractions.

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13. Integration by parts

14. Integration using complex exponential. Arc length of Cartesian and parametric curves.

First Order Ordinary Differential Equations

15. Definitions: ODE, order, general solution, initial value problems, linear/non-linear. Separable ODEs.

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16. Linear ODE using integrating factors.

17. Equilibrium points, stability.

18. GOOD FRIDAY/EASTER BREAK (nominal position)

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19. Applications: Doomsday and logistic population models with and without harvesting.

20. MID-SEMESTER TEST (nominal position)

21. Applications: Mixing problems with constant and variable volume.

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22. ANZAC DAY (nominal position)

23. Applications: Introduction to electric circuits. RC and LR series circuits. Transient and steady state solutions.

Second Order Ordinary Differential Equations

24. Definitions: homogeneous, inhomogeneous, linear/non-linear. Solution of homogeneous constant coefficient linear ODEs.

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25. Solution of inhomogeneous constant coefficient linear ODEs. Find particular solutions using method of undetermined coefficients.

26. Solution of inhomogeneous constant coefficient linear ODEs. Superposition of particular solutions.

27. Applications: Free vibrations of hanging spring-mass systems including air resistance.

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28. Applications: Forced vibrations of hanging spring-mass systems including air resistance. Resonance. [*Applications: LRC and LC series electric circuits. Resonance.]

Functions of Two Variables

29. Introduction to functions of two variables. Equation of plane in point-normal and Cartesian forms.

30. Level curves and cross sections. Sketching surfaces.

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31. First and second order partial derivatives. Chain rule for partial derivatives.

32. Gradient and directional derivatives. Steepest descent.

33. Tangent planes and normal lines. Linear approximations.

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34. Classification of stationary points of functions of two variables.

35. Partial integration of functions of two variables.

36. Double integrals over rectangular domains.

Notes: * indicates material which may be included if Anzac Day does not coincide with a lecture day and/or may omitted if time does not allow.
