

Subject code: 620-156

Subject name: Linear Algebra

Credit points: 12.5

Coordinator: Karen Baker

Semesters of offer: 2008: 2; 2009 onwards: 1, repeat 2, Summer

Prerequisites: Study score of 27 or more in VCE Specialist Mathematics 3/4 or equivalent, or one of [07]620-151, 620-154, [07]620-161 or permission from the Director of the Mathematics and Statistics Learning Centre

Mode of delivery: Lectures, tutorials and computer labs

Contact hours: 36 one-hour lectures (three per week), 11 one-hour tutorials (one per week), 11 one-hour computer laboratory classes (one per week)

Estimated total time commitment: 120 hours

Description: This subject gives a solid grounding in key areas of modern mathematics needed in science and technology. It develops the concepts of vectors, matrices and the methods of linear algebra. Students should develop the ability to use the methods of linear algebra and gain an appreciation of mathematical proof. Little of the material here has been seen at school and the level of understanding required represents an advance on previous studies.

Systems of linear equations, matrices and determinants; vectors in \mathbb{R}^n , cross product, scalar triple product, lines and planes; vector spaces, linear independence, basis, dimension; linear transformations, eigenvalues, eigenvectors; inner products, least squares estimation, symmetric and orthogonal matrices.

Assessment: Up to 25 pages of written assignments 10% (due during semester), two 45-minute written computer laboratory tests 10% (held during semester), a 3-hour written examination 80% (in the examination period).

Prescribed texts: H. Anton and C. Rorres, *Elementary Linear Algebra, Applications Version*, 9th edition, Wiley, 2005.

Notes: Students may only gain credit for one of [07]620-122, [08]620-142, 620-156, 620-157, [05]620-192, [05]620-194 or [07]620-211.

Students in the combined degree BE/BSc should note that credit exclusions exist between this subject and Engineering mathematics subjects. Refer to entries for [07]431-201 Engineering Analysis A and [07]431-202 Engineering Analysis B for details.

Subject objectives: Students completing this subject will:

- Use matrix techniques to represent and solve of a system of simultaneous linear equations;
- Understand the use of vectors in describing lines and planes in solid geometry;
- Understand the extension of vector concepts to abstract vector spaces of arbitrary finite dimension;
- Understand linear transformations, their matrix representations and applications;
- Become familiar with the use of a computer package for symbolic and numeric calculation.

Generic skills: In addition to learning specific skills that will assist students in their future careers in science, they will have the opportunity to develop generic skills that will assist them in any future career path. These include

- problem-solving skills: the ability to engage with unfamiliar problems and identify relevant solution strategies;
- analytical skills: the ability to construct and express logical arguments and to work in abstract or general terms to increase the clarity and efficiency of analysis;
- collaborative skills: the ability to work in a team;
- time management skills: the ability to meet regular deadlines while balancing competing commitments.
- computer skills: the ability to use an appropriate computing package.

Lecture-by-lecture outline:

Linear equations, Matrices and Determinants

1. Systems of linear equations. Coefficient arrays. Row operations.
 2. Reduction of systems to reduced row-echelon form.
 3. Consistent and inconsistent systems. Infinite sets of solutions with one, two or more parameters.
-
4. Properties of matrices. Row and column matrices. Special matrices (identity, zero, upper/lower triangular, diagonal, elementary, ...).
 5. Matrix algebra. Matrix inverses. Augmented matrices.
 6. Row equivalence. Inverses by row reduction. Rank.
-
7. Singular matrices. Cofactor expansion of determinants. Examples of 2×2 and 3×3 determinants.
 8. Row operations on determinants. Properties of determinants.

Euclidean Vector Spaces

9. Vectors in \mathbb{R}^n . Linear combinations of vectors in \mathbb{R}^n . Dependence and independence of vectors in \mathbb{R}^n .
-

10. Brief revision of vectors, dot product in \mathbb{R}^n . Cross product on \mathbb{R}^3 . Area of a parallelogram.
 11. Lines and planes. Intersection of lines with planes, lines with lines, planes with planes
 12. Scalar triple product. Determinant form. Volume of a parallelepiped. Applications such as distance of from a line to a plane.
-

13. Linear combinations, linear dependence/independence in \mathbb{R}^n .
 14. Planes through origin are closed under addition and scalar multiplication. Likewise the solution set to homogenous linear equations. Subspaces of \mathbb{R}^n .
 15. Subspaces of \mathbb{R}^n (continued), spanning sets.
-

16. Basis and dimension in \mathbb{R}^n .
 17. Solution space, nullity, column space, row space, rank.
 18. GOOD FRIDAY/EASTER BREAK (nominal position)
-

General Vector Spaces

19. General vector spaces, subspaces, eg. matrix/polynomial/DE with constant coefficients.
 20. Basis and dimension.
 21. ANZAC DAY (nominal position)
-

22. Mappings from \mathbb{R}^2 to \mathbb{R}^2 : reflections, compressions, expansions, shears, rotations.
 23. General linear transformations.
 24. Kernel and image, rank and nullity.
-

25. Matrix representation for transformations $\mathbb{R}^n \rightarrow \mathbb{R}^m$. Coordinates.
 26. Matrix representation in general.
 27. Change of basis/coordinates
-

Eigenvalues

- 28. Eigenvalues and eigenvectors of matrices, characteristic polynomial.
 - 29. Eigenspaces.
 - 30. Similarity transformations and diagonalisation. Matrix powers. Cayley-Hamilton Theorem (result, not proof)
-

Inner Products

- 31. Real inner product. Norm, distance, angle, Cauchy-Schwartz inequality.
 - 32. Orthogonality. Projections.
 - 33. Gram-Schmidt process. Orthonormal sets.
-
- 34. Least squares estimation. Curve fitting.
 - 35. Symmetric matrices, orthogonal matrices, and orthogonal diagonalisation.
 - 36. Revision
-

Notes: If Anzac Day (semester 1) does not fall on a lecture day, or in semester 2, an extra revision lecture will be included.
