Cost Optimisation of an Underground Mine

A mining company is planning to extract ore (say gold) from a deep underground orebody. The size, position and approximate volume of ore in the orebody are known from geological surveys. A method of extracting the ore has also been chosen. Extraction equipment will be sent into the orebody and ore will be removed via four access points at the base of the orebody. The positions of these access points have been determined and they are all at the same depth, exactly 300 meters below the surface. The ore will be taken to the surface via a system of tunnels (known as "drives") and a vertical shaft. The position on the surface of the top of the shaft is already known, and cannot be changed. You have been asked to design the network of drives connecting the four access points to the base of the shaft, so as to minimise the development and construction costs, that is the costs of building the tunnels and shaft.

PART (1):

In the first instance it has been decided by the mining company's engineers to sink the shaft to the same depth as the access points. This means that the drives will all be horizontal. The mining company wants you to optimise as far as you can the development and construction costs of the drives. For this part of the problem the cost of the shaft can be treated as a fixed amount and so does not need to be considered. There is a fixed cost of $2,000 per meter for building drives, and you can assume there is no added cost for junctions etc. How would you design the drives network so that the construction costs are minimum?

PART (2):

On discussing the problem with the mining company's engineers it becomes clear that the base of the shaft does not need to be as deep as the access points. Although the position of the top of the shaft is fixed, you realise that you may be able to make further savings by minimising the construction costs of the shaft plus the drives. The cost of building the shaft is about $10,000 per meter. There is also an added constraint: the vehicles that haul ore from the access points to the shaft cannot travel at a gradient greater than 1/8, that is, they cannot move upwards by more than a meter for each 8 meters they travel horizontally. Hence, 1/8 is the maximum gradient for any drive, though drives are allowed to zigzag or spiral in 3 dimensions in order to achieve any desired vertical displacement between two points. Can additional savings be made over the solution to Part (1) by choosing a different position for the base of the shaft? How would you design the network composed of drives and a shaft so that the overall construction costs are minimum?

Remember that your goal is to devise an effective and convincing method of finding the minimum cost - you are not expected to completely solve the problem this afternoon. If, however, you can apply your method to give a sketch of a reasonably good solution that would be advantageous. But ultimately what is really important is that you display a good understanding of the problem and find a feasible and mathematically convincing method of solving it.

Good Luck!
Problem Summary:

The orebody, the access points to the orebody and the base of the shaft are shown in the plan view in the attached figure. The problem has two parts:

(1) Assume the access points and the base of the shaft are all at the same depth. A network of drives must be built connecting the access points to the base of the shaft. How would you design the drives network so that the construction costs are minimum?

(2) Assume that the base of the shaft does not need to be at the same depth as the access points. Given that the maximum allowable slope of any drive is 1/8, can further savings be achieved by changing the depth of the shaft? How would you design the network of shaft and drives so that the construction costs are minimum?

Locations and Costs

Assume the x-axis points east and the y-axis points north and the z-axis points vertically upwards. Then in both parts of the problem the (x, y, z) coordinates in metres of the access points and the top of the shaft are:

| Access points | A1   | (4, 22, 0) |
|               | A2   | (45, 52, 0) |
|               | A3   | (92, 117, 0) |
|               | A4   | (138, 75, 0) |
| Shaft Top     | ST   | (98, 6, 300) |

Cost of building drives $2,000 per m.
Cost of building shaft $10,000 per m.

For Part (1) the position of the base of the shaft is

| Shaft Base | SB   | (98, 6, 0) |

For Part (2) the position of the base of the shaft is of the form (98, 6, k), where k lies between 0 and 300.
Plan view of the ore deposit, access points and shaft.