

# *Real world mathematics in action!*

## *I'd like to see that!*

Christine Mangelsdorf and Antoinette Tordesillas  
Department of Mathematics and Statistics  
The University of Melbourne

### 1. INTRODUCTION

To celebrate the International Year of Mathematics 2000, we organized a one-day maths fair for year 11-12 students and their teachers. The fair entitled “*Real world mathematics in action! I'd like to see that!*” was held on April 28, 2000, from 9am-5pm at the University of Melbourne. The fair was funded by the Department of Mathematics and Statistics of the University of Melbourne, National Science Week 2000, The Australian Mathematical Society, Apple and Analytica. Three hundred students and staff from thirty-one schools across Melbourne participated.

The objectives of the fair were twofold:

- to promote the importance and broad applications of mathematics and problem solving skills;
- to raise awareness of career opportunities for mathematics and statistics graduates.

Bearing in mind our predominantly young audience, we designed a program that would both engage as well as inform. This led to two key events: *Mathematicians Exposed!* and the *Maths in Industry and Technology (MIT) Challenge*. In the morning, all participants attended *Mathematicians Exposed!* while, in the afternoon, participants had the choice of competing in the *MIT Challenge* or attending the second session of *Mathematicians Exposed!*

The survey of the fair showed the event to be an overwhelming success with 91% of respondents saying that it was both enjoyable and informative. In particular, a number of Victoria's top ranking 2000 VCE students who participated in the fair have commented on how the event has influenced their choice of subject majors at University. The personal interactions with various staff in the Department, combined with the opportunity to meet graduates from such a wide range of areas and to learn about where their mathematical training has taken them, have served as a great inspiration to many students.

Apart from having accomplished our objectives, this event has clearly proven to be an effective means of recruiting the very best students; in fact, a number of these are now taking Mathematics at the University of Melbourne.

## 2. MATHEMATICIANS EXPOSED!

### Session 1 (9am-12.15pm)

This event was designed to showcase career profiles of mathematics and statistics graduates. Representatives from the areas of information technology, telecommunications, medicine, finance, defence and superannuation gave oral presentations on how and where mathematics and statistics are used in their daily work and the level of mathematical training needed to pursue careers in their profession. All of the speakers have international experience in their chosen fields. To illustrate the breadth and expertise of the speakers, we include the career profiles of three of the speakers. These career profiles were used to advertise the event.

Professor Michael Barnsley, a Professor of Mathematics, is the co-founder and former CEO of Iterated Systems Inc. The company was created to make the discovery of the automatic fractal transform process for image coding into a viable business. As CEO, Professor Barnsley was responsible for raising the revenue of the company from \$1 million in 1991 to \$6.8 million in 1993. Iterated Systems Inc. was successful in the CD-ROM imaging market, and its technology was used in Microsoft Encarta and many other commercial titles.

Dr John Carlin is the Deputy Head of the Clinical Epidemiology and Biostatistics Unit at the Murdoch Children's Research Institute, Royal Children's Hospital. Dr Carlin has worked in health and medical research at the Royal Children's Hospital since 1991, after doing Honours in mathematics and statistics and a PhD in statistics. He is involved in a wide range of projects concerned with improving the clinical care of sick children and preventing the development of diseases, injuries and unhealthy behaviours. Sound statistical thinking and analytic methods are crucial in solving the real-world problems of modern medical and public health research.

Dr Maris Ozols is a Senior Research Scientist at DSTO (Defence Science and Technology Organisation). Maris completed a PhD in pure mathematics. Since 1990 he has worked in various aspects of trusted systems at DSTO, including system safety, formal verification techniques, and information security. Since 1998 he has been leading a section involved with public key cryptography, secure messaging, and formal modelling of information security systems. In 1999 he lectured at the University of South Australia on "High Integrity Systems".

The final talk in this session was an interactive presentation on the art of mathematical modelling. The aim of this presentation was to basically "kick-start" the *MIT Challenge* as the students were engaged in the mathematical modelling of a simple real world problem and were provided with ideas and techniques they could use in this competition. Specifically, the problem that was discussed was that of minimising the fuel consumption for a train travelling on a flat track between two stations.

### Session 2 (1.45pm-4pm)

The afternoon session of *Mathematicians Exposed!* was aimed at students who did not wish to compete in the *MIT Challenge* but who wanted to try their hand at solving short maths

problems and meet people who use mathematics in their jobs. The session was divided into two parts: a problem solving competition and exhibits.

**Analyze this!:** Whilst teachers were involved in their own 1-hour workshop, students competed in a 1-hour problem solving competition *Analyze this!* organized by the Melbourne University Mathematics and Statistics Student Society. Around 40 teams, each consisting of 3 students, competed. Teams were asked a series of short mathematics problems and mathematics trivia questions. Although most students coped with the problem-solving exercises, many needed to brush up on their maths trivia with questions ranging from "What was the name of Newton's dog?" to "How many zeros are there in a googol and googolplex?"<sup>1</sup> The winning teams from Korowa Anglican Girls School and Melbourne Grammar School received movie passes and certificates.

**Research exhibits:** There was a wide range of interactive exhibits and displays covering research projects that have been or are currently being conducted by staff from the Department of Mathematics and Statistics including the Statistical Consulting Centre. The displays included projects on: cooking of wheat grains for the manufacture of Vita-Brits (Uncle Tobys); effectiveness of a new flu treatment (Glaxo Wellcome); management of mine wastes (BHP); pavement design (ARRB Transport Research); estimating the cost of phone services to rural Australia (Australian Communications Authority); off-road vehicle design simulations (United States Army); knots; nano-technology and operations research. These were accompanied by exhibits from zoology, earth sciences, chemistry and meteorology which highlight the use of mathematics in these areas as well as information about careers for mathematics graduates in consulting, finance, superannuation and the Australian defence force.

### 3. MATHS IN INDUSTRY AND TECHNOLOGY (MIT) CHALLENGE

Forty teams consisting of four year 11-12 students competed in the *Mathematics in Industry and Technology (MIT) Challenge*. The *MIT Challenge* was designed to be a "mini version" of the Mathematics in Industry and Technology Study Group (MISG) which meets once a year to "brainstorm" on industry problems; the MISG is comprised of academics and representatives from various industries and government agencies in Australia and New Zealand.

Our main objective for the *MIT Challenge* was to give students a taste of real world consulting in the format of a competition. With this in mind we designed a team competition that would engage students in all aspects of consulting: meeting the client and understanding the client's problem, working together as a team, discussing ideas, planning a solution strategy, communicating ideas to the client in both written and oral presentations. The format of the competition was as follows: an industry representative ("the client") presents a real world problem for the teams ("the consultants") to establish a solution *approach* within 3 hours. The teams then submit a written report of their proposed solution approach to a panel of judges from industry and academia. The judges evaluate the written reports, selecting the top 6 ranked teams. A representative from each of the 6 top ranked teams gives a 5-minute oral presentation of their team's findings. The actual program for the *MIT Challenge* was:

---

<sup>1</sup> Answers: Diamond was the name of Newton's dog. A googol is 1 followed by 100 zeros; a googolplex is 10 to the power of a googol.

12:20-12:50 PM	<i>MIT Challenge</i> problem presentation
1:00-3:30 PM	Teams work on <i>MIT Challenge</i> problem
3:30-4:00 PM	Short-listing of teams (afternoon tea)
4:00-4:35 PM	<i>MIT Challenge</i> student presentations
4:35-4:50 PM	<i>MIT Challenge</i> solution presentation

The student presentations and solution presentation provided an important closure to the *MIT Challenge*, as competing teams were able to see how close they and other teams had come to the optimal solution. The final winners were announced a week after the fair to give judges time to examine the written reports in detail. Prizes were awarded according to the quality of the proposed written solution and oral presentation. Teams from the following schools won the major prizes in the *MIT Challenge*:

**1st Prize:** Caulfield Grammar School (\$800 for team)

**2nd Prize:** Mt Scopus Memorial College (\$400 for team)

**3rd Prize:** Parade College (\$200 for team)

Other short listed teams were awarded certificates of merit.

A key aspect of organising the *MIT Challenge* was to find a real life problem that was accessible to year 11 mathematics students so that they could make a reasonable attempt at solving the problem in under 3 hours. The *MIT Challenge* problem was developed by Dr. Marcus Brazil (Department of Electrical and Electronic Engineering, University of Melbourne). The problem stems from a research project being undertaken by him and his collaborators from the University of Melbourne and the Australian Mining Consultants for Western Mining Corporation. The *MIT Challenge* problem was presented by Mr. Brian Hall, a Principal Mining Engineer from the Australian Mining Consultants. Mr Hall gave a broad overview of the mining industry, discussing where mathematics is used in the mining industry and then zoomed in on the particular problem the teams would be considering in the *MIT Challenge*. The following sections give a summary of the *MIT Challenge* problem and possible solution strategies for this problem.

### 3.1 The MIT Challenge problem

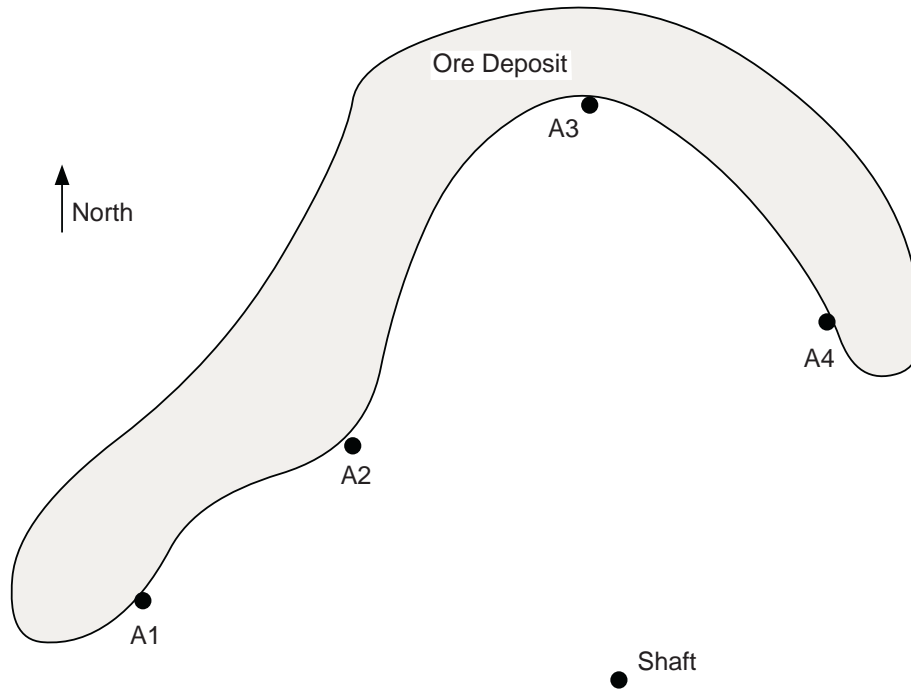
The goal of this problem is to devise an effective and convincing *method* of finding the minimum cost of an underground mining network. The ore body, the access points to the ore body and the base of the shaft are shown in Figure 1. The problem has two parts:

**Part (1):** Assume that the access points and the base of the shaft are at the same depth. A network of drives must be built connecting the access points to the base of the shaft. How would you design the drives network so that the construction costs are minimized?

**Part (2):** Assume that the base of the shaft does not need to be at the same depth as the access points. Given that the maximum allowable slope of any drive is  $1/8$ , can further savings be achieved by changing the depth of the shaft? How would you design the network of shaft and drives so that the construction costs are minimized?

*Given information on locations and costs:* Assume the  $x$ -axis points east and the  $y$ -axis points north and the  $z$ -axis points vertically upwards. Then in both parts of the problem the  $(x,y,z)$  coordinates in metres of the access points and the top of the shaft are:

Access points: A1 (4, 22, 0) A2 (45, 52, 0) A3 (92, 117, 0) A4 (138, 75, 0)  
 Shaft Top: ST (98, 6, 300)  
 Cost of building drives: \$2,000 per metre  
 Cost of building shaft: \$10,000 per metre  
 For Part (1), the position of the base of the shaft is (98, 6, 0).  
 For Part (2), the position of the base of the shaft is (98, 6,  $k$ ), where  $0 \leq k \leq 300$ .



**Figure 1:** Plan view of the ore body showing the access points to the ore body relative to the base of the shaft.

### 3.2 Solution to the MIT Challenge problem - Part (1)

The network required for Part (1) is an example of a *Steiner minimum network* - a network connecting a given set of points in a plane such that the total length of edges in the network is as small as possible. Such networks can contain extra junctions, which are referred to as *Steiner Points*. The main properties of a Steiner minimum network are:

- All edges are straight lines
- The network contains no circuits (or cycles)
- No two edges meet at an angle of less than 120 degrees
- Exactly 3 edges meet at any Steiner point and the angle between each pair of them is exactly 120 degrees
- There are at most  $n-2$  Steiner points, where  $n$  is the number of fixed nodes given in the original problem ( $n=5$  in this problem)
- There are many possible *topologies* (or patterns of connections).

The key question is how to construct such a minimum network, for a given set of nodes. Two approaches to this aspect of the problem are as follows:

- *Exact Methods*

Example 1: Set up a system of equations whose variables are the coordinates of the Steiner points, and minimise the lengths of the edges.

Example 2: Use the geometric properties of the network to solve exactly.

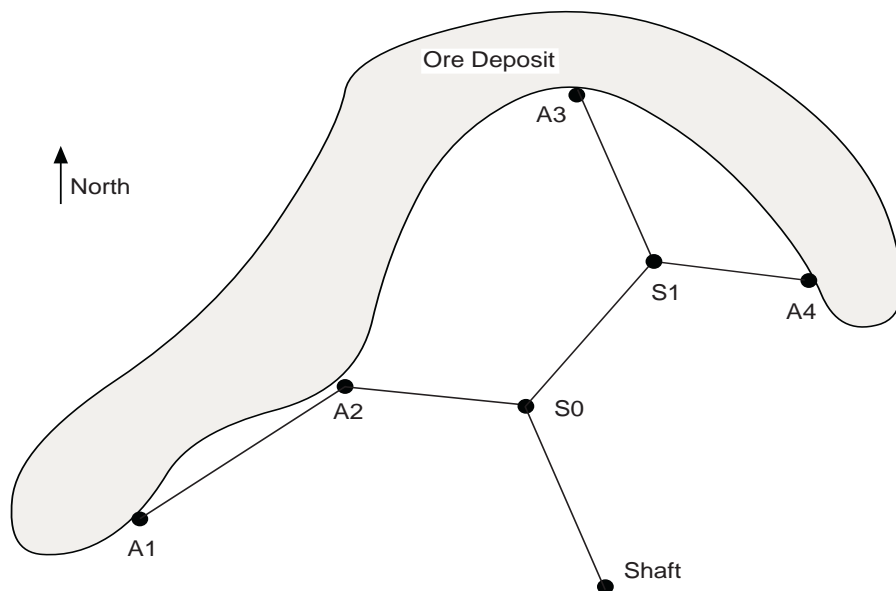
- *Iterative Methods*

Example 1: Start with an approximate solution, look for Steiner points with angles  $< 120$  degrees, move each such Steiner point a small distance into the small angle to make it larger. Repeat until all angles are close to 120 degrees.

The answer to Part (1) is as follows:

- Junctions:  $s_0 = (81.5, 46.8, 0)$ ,  $s_1 = (107.1, 79.4, 0)$
- Length of Drives network: 244.9 m
- Cost of Drives: \$489,725
- Total cost (including shaft \$3M): \$3,489,725

A plan view of the optimal drives network for Part (1) is shown in Figure 2.



**Figure 2:** A plan view of the optimal drives network for Part (1)

### 3.3 Solution to the MIT Challenge problem - Part (2)

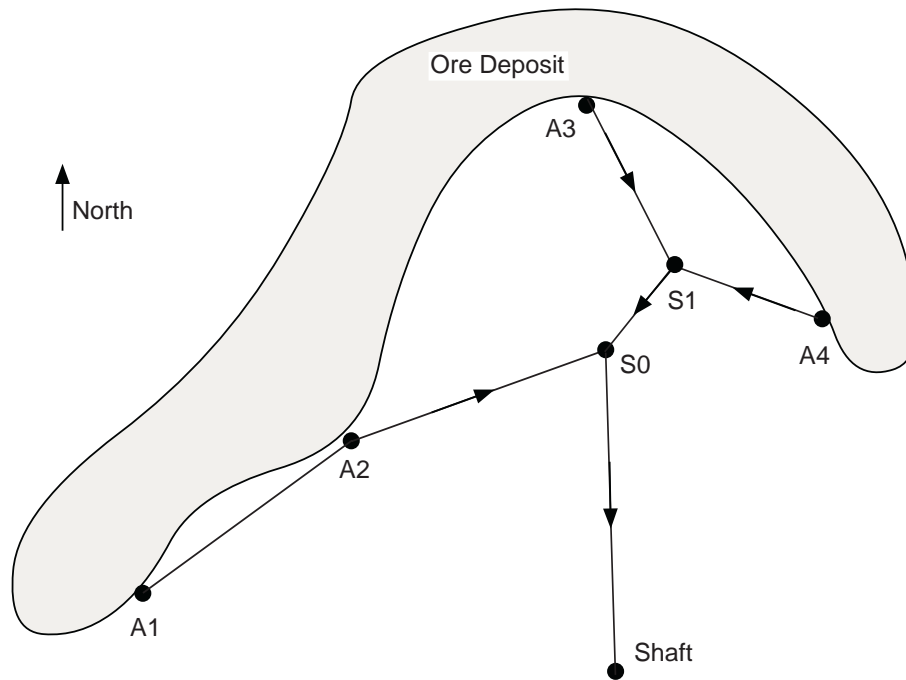
The second part is much more difficult. In particular, the geometric properties involving 120 degree angles no longer apply. It is useful to note that: to go up 1 metre by shaft costs \$10,000, but to go up 1 metre by a (zigzag) drive costs \$16,000. This suggests that we should never use zigzag drives in the optimal solution. The shaft is sufficiently expensive so that savings can be made by raising the base of the shaft. In essence, this part involves lifting the shaft base as much as possible without creating any zigzag drives.

The optimal solution was obtained using a computer program employing an iterative algorithm similar to that described in Part (1). The added difficulty in devising such an algorithm for this part is that it is much harder to decide what direction to move the Steiner points to improve a given approximate solution. The optimal solution is:

- Junctions:  $s_0 = (92.5, 73.6, 6.5)$ ,  $s_1 = (105.9, 86, 4.25)$

- Shaft base: (98, 6, 15)
- Length of Drives network: 58.6 m
- Cost of Drives: \$517,121
- Total cost (including shaft): \$3,367,121

A plan view of the optimal solution for Part (2) is shown in Figure 3. The drives with arrows pointing in the upward direction have a slope of exactly  $1/8$ . The drive between A1 and A2 is horizontal.



**Figure 3:** A plan view of the optimal solution for Part (2)

The teams were given access to basic computing facilities during the *MIT Challenge* so some used spreadsheets to help with calculations. In a couple of hours, competing teams of students came up with practical solutions that give huge savings on tunnelling costs and devised effective strategies for determining optimal tunnel layouts. Some of the student oral presentations were of an extremely high standard as was the mathematical insight displayed in the written reports. Many of the teams achieved solutions that were close to the optimal solution for Part (1). As expected, few teams made any real progress in Part (2).

#### 4. CONCLUSION

A survey was given to students on registration and was collected at the end of the fair. The survey results show that the fair was an overwhelming success with 91% of respondents saying they enjoyed the event. Some of the comments were:

- *"It gave me a wider range of knowledge of how maths is used in industry. It's made me reconsider my career path."*
- *"The day is a fantastic concept."*

- *"I learnt plenty. In particular, about lateral thinking."*
- *"fun and useful"*
- *"informative and enjoyable"*
- *"The main benefit, besides the maths, was getting students into the University environment and giving them time to look around."*

At least two of Victoria's top students, both of whom won prizes in the *MIT Challenge*, have changed their career paths as a result of their participation in the fair, and are now pursuing a Bachelor of Science with a major in mathematics at the University of Melbourne.

The survey results showed that the *MIT Challenge* was the highlight of the fair. All teams enjoyed the *MIT Challenge* finding it *"thought provoking, challenging and fun"*. The format of the *MIT Challenge* worked extremely well and all aspects were considered essential by the competing teams.

Session 1 of *Mathematicians Exposed!* was met with mixed responses. Some of the speakers and displays were excellent but some of the talks were unsuitable for year 11-12 students. In general, participants felt that two sessions of talks were too long to concentrate and remain seated in one place; one session of talks, displays and a tour of the University would be ideal. In future, we are considering changing the format and length of *Mathematicians Exposed!* in order to shorten the whole fair and keep students actively engaged.

Session 2 of *Mathematicians Exposed!* was attended by about 140 students and teachers. Those attending the displays found them to be enjoyable and informative. Most of the teams found the problem solving competition *Analyze this!* too challenging, albeit enjoyable. The general impressions were that the trivia questions were too hard and not linked to the theme "Maths in Action" and that the mathematical questions were too difficult for the time allowed.

A one-and-a-half hour video showing the highlights of the Maths Fair was produced and is available from the authors.

Overall, the inaugural Maths Fair was a great event, especially for recruiting the best students to take up Science at University. Plans are now being made to hold the event bi-annually. We look forward to seeing more students and teachers in the upcoming 2002 Maths in Action!

## **ACKNOWLEDGEMENTS**

We thank the Department of Mathematics and Statistics of the University of Melbourne, National Science Week 2000, The Australian Mathematical Society, Apple and Analytica for their financial support of the Maths Fair. Without their support the event would not have been possible.

We gratefully acknowledge the efforts of Dr Marcus Brazil in designing and presenting the solution to the *MIT Challenge* problem and for leading the judging panel. We also thank all the industry representatives who participated in this event, in particular, Mr Brian Hall for giving an informative talk on the mining industry and for presenting the *MIT Challenge* problem.