Department of Mathematics and Statistics

620–231 Vector Analysis

Semester 1, 2006

Subject Organisation

1 Syllabus

This subject develops the manipulation of partial derivatives and vector differential operators. Students should develop the ability to obtain extrema of functions of several variables, calculate line, surface and volume integrals, and to work in curvilinear coordinates. This subject demonstrates the fundamental concepts of vector calculus and the relations between line, surface and volume integrals.

Specific topics include: *Functions of several variables*: limits, continuity, differentiability, matrix version of chain rule, Jacobian, Taylor polynomials, Lagrange multipliers. *Vector calculus*: vector fields, flow lines, curvature, torsion, gradient, divergence, curl and Laplacian. *Integrals over paths and surfaces*: line, surface and volume integrals; change of variables; applications including averages, moments of inertia, centre of mass. Green's theorem, Divergence theorem, Stokes' theorem; general curvilinear coordinates.

2 Prerequisites

The prerequisites for 620-231 are either one of 620-113, 620-123, 620-143, [2005]620-193.

3 Lectures, Tutorials and Consulting Times

The lecturer is Dr. Richard Brak – room 197 in the Richard Berry Building.

There are 35 lectures (three per week - excluding Good Friday). The lectures are on Monday, Wednesday and Friday at 12pm in Elisabeth Murdoch-Theatre A.

There are 11 one-hour tutorials (one per week). Tutorials will be assigned, and the list will appear on the Second Year Notice Board. They should start in the second week of semester.

Additional tutorial assistance available will be indicated on the Second Year Notice Board.

In addition to lectures and tutorials Dr. Brak is available for consultation at the following times: Monday, Wednesday and Friday 3.15pm-4.15pm.

4 Problem Sheets

There are five problem sheets. The problems can be grouped into two types:

• Questions labelled **Revision** cover material that is assumed knowledge from 620-121, 620-123, 620-141 and 620-143. You should be able to complete all of these questions.

• Questions that do not have a label **Revision** are the core questions for 620-231. It is essential that you attempt **all** of these problems. These questions form the examinable material for 620-231.

5 Assessment

The assessment is composed of three parts:

• A three hour exam at the end of semester;

• A mid-semester test on Wednesday 5th April which will take place a *12pm in Wilson Hall*

• Two assignments due as follows:

(1) due 10.30 am on Monday 27th March;

(2) due 10.30 am on Monday 8th May;

Each piece of assessment is **compulsory**.

The **Final Mark** in 620-231 is computed as:

Final Mark = 80% Exam + 8% Assignments + 12% Mid-semester test

Note:

• The assignments will be handed out in the first week of lectures and will also be put on the web.

• Make sure that you write your name, student number, tutors' name and tutorial time on each assignment.

• **Late assignments will not be accepted.**

• Assignment 1 must be submitted with a completed plagerism cover sheet. This plagerism cover sheet will apply to all assignments in 620-231 during semester 1. Plagerism cover sheets will be handed out in lectures and will also be available on the web.

• **The assignments will not be marked until a completed plagerism cover sheet is submitted.**

• Any medical certificates relating to the assignments should be given to Dr. Brak.

6 Text Books

There are three textbooks recommended for extra reading and problems. They are:

• ***Vector Calculus***, Fourth Edition, by J. E. Marsden and A. J. Tromba, published by Freeman.

• ***Calculus of Several Variables***, Fourth Edition, by R. A. Adams, published by Addison-Wesley.

• ***Calculus Concepts and Contexts***, 2nd edition by J. Stewart, published by Brooks/Coles, Thomson Learning. This was the textbook used in 620-141 and 620-121.

All books are on reserve in the Mathematics Library and Baillieu Library, and may be purchased from the Book Room.

7 Lecture Notes

A copy of the lecture notes containing the theoretical content of the course is available for purchase. The examples that will be covered during the course are stated but the working is not given - a blank space in the booklet is avaible to fill in the working.

8 Web

Additional material for the subject can be found on the subject web site. Navigate to the 620-231 web page via the departments home page `www.ms.unimelb.edu.au`

9 References to Texts

This table gives references to the relevant section(s) of the text books ***Vector Calculus, 4th edition*** by Marsden and Tromba (labelled Marsden) and ***Calculus of Several Variables, 4th edition*** by Adams (labelled Adams) and to corresponding questions on the problem sheets.

Topic	**Marsden**	**Adams**	**Questions**
Functions of Several Variables			
Sketching surfaces revision	§2.1	§1.5, §3.1	53-54
Limits and continuity	§2.2	§3.2	1-5
Partial differentiation revision	§2.3, §3.1	§3.3, §3.4	6-7
Differentiability	§2.3	§3.6	8, 10
Tangent planes revision	§2.3, §2.6	§3.3	9
Matrix version of chain rule	§2.5	§3.6	11-12
Jacobian	pg 359	§3.8	13-14
Taylor polynomials	§3.2	§3.9	15-16
Extrema, constrained extrema	§3.3	§4.1, §4.2	17
Lagrange multipliers	§3.4	§4.3	18-20
Space Curves and Vector Fields			
Vectors revision	§1.1, §1.2, §1.3	§1.2, §1.3	21-23
Parametric paths - velocity, acceleration	§2.4, §4.1	§2.1	24-27
Arc length	§4.2	§2.3	28
Tangent vectors, curvature, torsion	pg 263-264	§2.4, §2.5	29-31
Vector fields	§4.3	§6.1	32
Flow lines	§4.3	§6.1	33-34
Differential operators	§4.4	§7.1	35-38, 41-45
Basic identities of vector analysis	§4.4	§7.2	41-45
Scalar potentials	pg 497-500	§7.2	39-40
Double and Triple Integrals			
Double integrals	§5.1, §5.2, §5.3	§5.1, §5.2	47-51
Areas and volumes using double integrals	§5.3	§5.2	48
Change of order of integration	§5.4	§5.2	50-51
Triple integrals	§5.6	§5.5	52
Elementary regions	§5.6	§5.5	55
Volumes using triple integrals	§5.6	§5.6	56, 62
Polar, cylindrical and spherical coordinates	§1.4	§5.4, §5.6	57-58
Change of variables for multiple integrals	§6.2	§5.4, §5.6	59-64
Averages, centre of mass, moment of inertia	§6.3	§5.7	63-64

Topic	Marsden	Adams	Questions
Integrals over Paths and Surfaces			
Path integrals	§7.1	§6.3	66-67
Line integrals	§7.2	§6.4	68-70
Parametrization of paths and surfaces	§7.3	§6.4, §6.5	65, 71
Tangent planes to parametrized surfaces	§7.3	§6.5	72-73
Area of a surface	§7.4	§6.5	74
Integrals of scalar functions over surfaces	§7.5	§6.5	75-76
Integrals of vector functions over surfaces	§7.6	§6.6	77-78
Integral Theorems			
Green's theorem	§8.1	§7.3	79-82
Divergence theorem in plane	§8.1	§7.3	83-84
Stokes' theorem	§8.2	§7.5	85-87
Conservative fields	§8.3	§6.2	88-89
Gauss' divergence theorem	§8.4	§7.4	90-92
Mixed integral theorems	§8.1-§8.4	§7.3-§7.5	93-94
General Curvilinear Coordinates			
Orthogonal curvilinear coordinates	None	§7.7	95-98
Differential operators	None	§7.7	95-98

This table gives references to the relevant section(s) of the text book ***Calculus Concepts and Contexts, 2nd edition*** by James Stewart (labelled Stewart) and to corresponding questions on the problem sheets.

Topic	**Stewart**	**Questions**
Functions of Several Variables		
Sketching surfaces revision	§9.6	53-54
Limits and continuity	§2.2, §2.3, §2.4, §11.2	1-5
Partial differentiation revision	§11.3	6-7
Differentiability	§11.4	8, 10
Tangent planes revision	§11.4	9
Matrix version of chain rule	None	11-12
Jacobian	pg 904	13-14
Taylor polynomials	§8.7	15-16
Extrema, constrained extrema	§11.7	17
Lagrange multipliers	§11.8	18-20
Space Curves and Vector Fields		
Vectors revision	§9.2, §9.3, §9.4	21-23
Parametric paths - velocity, acceleration	§10.2, §10.4	24-27
Arc length	§10.3	28
Tangent vectors, curvature, torsion	§10.3	29-31
Vector fields	§13.1	32
Flow lines	pg 924	33-34
Differential operators	§13.5	35-38, 41-45
Basic identities of vector analysis	§13.5	41-45
Scalar potentials	§13.3	39-40
Double and Triple Integrals		
Double integrals	§12.1, §12.2, §12.3	47-51
Areas and volumes using double integrals	§12.2, §12.3	48
Change of order of integration	§12.3	50-51
Triple integrals	§12.7	52
Elementary regions	§12.7	55
Volumes using triple integrals	§12.7	56, 62
Polar, cylindrical and spherical coordinates	§9.7	57-58
Change of variables for multiple integrals	§12.4, §12.8, §12.9	59-64
Averages, centre of mass, moment of inertia	§12.5, §12.7	63-64

Topic	Stewart	Questions
Integrals over Paths and Surfaces		
Path integrals	§13.2	66-67
Line integrals	§13.2	68-70
Parametrization of paths and surfaces	§1.7, §10.5	65, 71
Tangent planes to parametrized surfaces	pg 787	72-73
Area of a surface	§12.6	74
Integrals of scalar functions over surfaces	§13.6	75-76
Integrals of vector functions over surfaces	§13.6	77-78
Integral Theorems		
Green's theorem	§13.4	79-82
Divergence theorem in plane	pg 958	83-84
Stokes' theorem	§13.7	85-87
Conservative fields	§13.3	88-89
Gauss' divergence theorem	§13.8	90-92
Mixed integral theorems	§13.4, §13.7, §13.8	93-94
General Curvilinear Coordinates		
Orthogonal curvilinear coordinates	None	95-98
Differential operators	None	95-98

10 Topics Outline

This is subject to small variation without notice.

Topic	Sheet
Functions of Several Variables	
Limits of functions of several variables	1
Continuity, differentiability of functions of several variables	1
Matrix version of chain rule, Jacobian	1
Taylor polynomials for functions of several variables	1
Extrema, stationary points, constrained extrema	1
Lagrange multipliers	1
Space Curves and Vector Fields	
Lagrange multipliers, parametric paths - velocity, acceleration	2
Arc length, tangent and normal vectors, curvature, torsion	2
Vector fields, flow lines, differential operators: grad	2
Differential operators: div, curl, Laplacian	2
Laplacian, vector analysis identities, scalar potentials	2
Using the vector analysis identities	2
Double and Triple Integrals	
Double integrals over rectangular domains	3
Double integrals over general domains, changing order of integration	3
Triple integrals, elementary regions	3
Triple integrals - examples, volumes	3
Volumes, polar and cylindrical coordinates	3
Spherical coordinates, change of variables for multiple integrals	3
Change of variables for multiple integrals	3
Change of variables for multiple integrals, averages, centre of mass	3
Integrals over Paths and Surfaces	
Moment of inertia, path integrals, line integrals	4
Line integrals, parametrization of paths and surfaces	4
Normals, tangent planes to parametrized surfaces, surface area	4
Integrals of scalar functions over surfaces	4
Integrals of scalar and vector functions over surfaces	4
Integrals of vector functions over surfaces, Green's theorem	4
Integral Theorems, General Curvilinear Coordinates	
Green's theorem in plane	5
Area via line integrals, Divergence theorem in plane	5
Stokes' theorem	5
Stokes' theorem, conservative fields	5
Conservative fields, Gauss' divergence theorem	5
Gauss' divergence theorem, general curvilinear coordinates	5
General curvilinear coordinates	5