620-231 Semester 1, 2004 Exam Solutions

Q1 (a) $\frac{\partial f}{\partial y} = \frac{9x^2y^2 + 6y^4 + 4yx^2}{(x^2+2y^2)^2}$

(b) $3/2$

(c) The limit does not exist since limits are different as we approach $(0,0)$ along $x=0$ and $y=0$.

(d) $f(x,y)$ is not continuous at $(0,0)$ since the limit does not exist as $(x,y) \to (0,0)$

Q2 (a) Maximum of $f = 3$ at $(1,1)$, $(-1,-1)$
Minimum of $f = 1$ at $(1,-1)$, $(-1,1)$.

(b) Global maximum is $f = 3$ at $(1,1)$, $(-1,-1)$
Global minimum is $f = 0$ at $(0,0)$

Q3 (a) $K = \frac{\sqrt{2}}{4}$

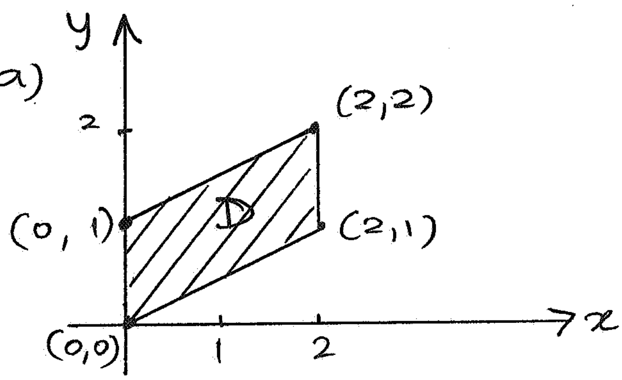
(b) $2\int_0^{\pi/2} (2t + \sqrt{2}\sin t)\, dt = \frac{\pi^2}{2} + 2\sqrt{2}$

Q4 (a) Proof – using properties of vectors.

(b) (i) $\underset{\sim}{0}$

(ii) $(6+4r^2)e^{r^2}$.

Q5 (a)



(b) $J = \frac{1}{2}$

$$I = \int_0^2 \int_0^2 u^3 v e^{v^2} \tfrac{1}{2}\, du\, dv$$
$$= e^4 - 1$$

Q6 (a) 480π

(b) $\frac{180}{13}$

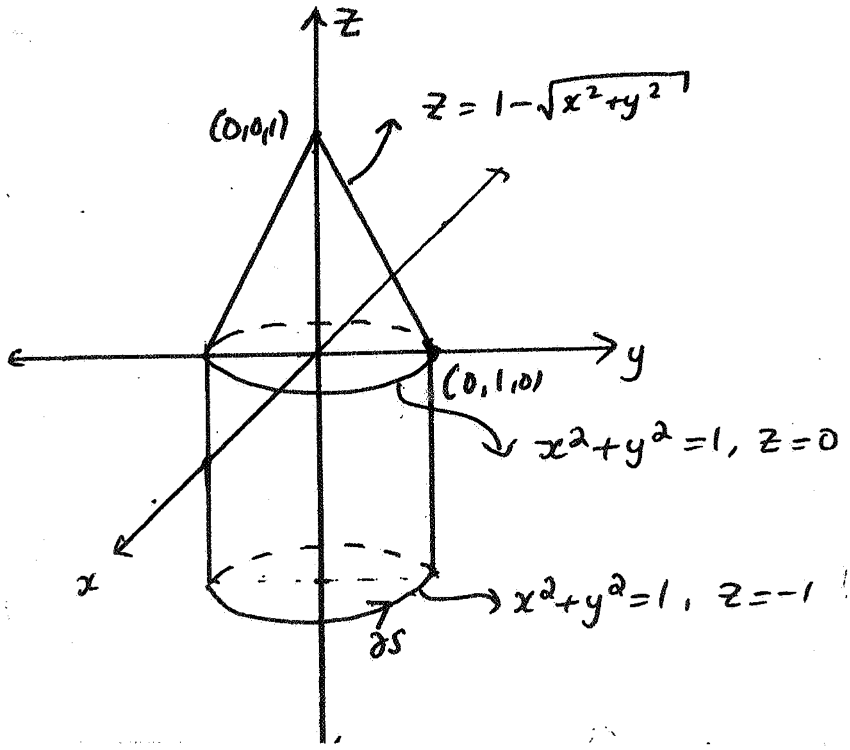
Q7 (a) Normal $= u^2 \sin v\,\underline{i} + u^2 \cos v\,\underline{j} - 2u^3\,\underline{k}$

(b) $x + 2z + 1 = 0$

(c) $\int_0^2 \int_{-3}^3 (u+v)\, u^2 \sqrt{1+4u^2}\, du\, dv$

Q8 $-11/30$.

Q9 (a)



(b) 0.

Q10 $84\pi = 48\pi + 36\pi + 0$ (3 surfaces)

Q11 Proof required.