22.4 Insurance risk

This is a simplified version of two common problems faced by insurance companies: calculating the probability that they go bust and estimating how much money they will make.

Suppose that an insurance company has current assets of $1,000,000. They have \( n = 1,000 \) customers who each pay an annual premium of $5,500, paid at the start of each year. Based on previous experience, it is estimated that the probability of a customer making a claim is \( p = 0.1 \) per year, independently of previous claims and other customers. The size \( X \) of a claim varies, and is believed to have the following density, with \( \alpha = 3 \) and \( \beta = 100,000 \),

\[
f(x) = \begin{cases} 
\frac{\alpha \beta^\alpha}{(x + \beta)^{\alpha+1}} & \text{for } x \geq 0, \\
0 & \text{for } x < 0.
\end{cases}
\]

(Such an \( X \) is said to have a Pareto distribution, and in the real world is not an uncommon model for the size of an insurance claim.)

We consider the fortunes of the insurance company over a five-year period. Let \( Z(t) \) be the company’s assets at the end of year \( t \), so

\[
Z(0) = 1,000,000,
\]

\[
Z(t) = \begin{cases} 
\max\{Z(t - 1) + \text{premiums} - \text{claims}, 0\} & \text{if } Z(t - 1) > 0, \\
0 & \text{if } Z(t - 1) = 0.
\end{cases}
\]

Note that if \( Z(t) \) falls below 0 then it stays there. That is, if the company goes bust then it stops trading.

22.4.1 Simulating \( X \)

Let \( X \) be the size of a typical claim as above. Calculate the cdf \( F_X \), \( \mathbb{E}X \), and \( \text{Var}X \).

Using the inversion method, write a subroutine to simulate \( X \).

Use simulation to estimate the pdf of \( X \) and compare your estimate to the true pdf. Your answer should include a plot like Figure 22.6.

22.4.2 Simulating \( Z \)

Write a function to simulate the assets of the company over five years, then use it to plot the assets as a graph like Figure 22.7.

Using your function, estimate:

1. The probability that the company goes bust, and
2. The expected assets at the end five years.
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Figure 22.6 Simulated pdf for insurance risk example.

Figure 22.7 Simulated assets for insurance risk example.
22.4.3 Profit taking

Suppose now that the company takes profits at the end of each year. That is, if \( Z(t) > 1,000,000 \) then \( Z(t) - 1,000,000 \) is paid out to the shareholders. If \( Z(t) \leq 1,000,000 \) then the shareholders get nothing that year.

Using this new scheme, estimate

1. The probability of going bust.
2. The expected assets at the end of five years, and
3. The expected total profits taken over the five years.

Compare these answers with your answers for Part 22.4.2 and comment.