1. Let $X = \{a, b, c\}$, and define $T := \{\emptyset, \{b\}, \{a, b\}, \{b, c\}, X\}$

   (a) Show that $T$ defines a topology on $X$.
   (b) Is $X$ connected?
   (c) Is $X$ compact?
   (d) Is $X$ Hausdorff?
   (e) Is $\{b\}$ closed? Isn’t that kind of weird?

2. Let $X \subseteq \mathbb{R}^2$ be the subspace $X := \{(x, y) \mid |y| \leq 1\}$

   and say that $(x, y) \sim (x + n, (-1)^n y)$ for every $n \in \mathbb{Z}$.

   (a) Describe $X$.
   (b) Check that $\sim$ is an equivalence relation.
   (c) What familiar object is the quotient space $X/\sim$?

3. Show that if $X$ is a finite set, and $T$ any topology on $X$, then $X$ is compact (in the topology $T$).

4. Show that if $f : X \to Y$ is a homeomorphism, then $U \subseteq X$ is open if and only if $f(U)$ is open.

   Conclude that $f$ sets up a bijection between the topology of $X$ and the topology of $Y$. Use this in the next problem.

5. Show that if $X$ and $Y$ are homeomorphic, then
   
   (a) $X$ is compact if and only if $Y$ is compact.
   (b) $X$ is Hausdorff if and only if $Y$ is Hausdorff.

6. Is $\mathbb{Q}$ compact (with the subspace topology from $\mathbb{Q} \subseteq \mathbb{R}$)?

7. True or false: If $X$ contains a non-compact subspace, then $X$ is not compact.

8. Show that if $X$ is compact, and $Y \subseteq X$ is closed, then $Y$ is compact.

9. Assume that $X$ is compact, and $f : X \to Y$ is continuous. Let $f(X) = \{f(x) \mid x \in X\} \subseteq Y$ be the image of $X$ in $Y$ under $f$.

   (a) Show that $f(X)$ is compact if $X$ is compact.
   (b) Show that $f(X)$ is connected if $X$ is connected.

10. Use the previous problem to show that if $X$ is compact, and $\sim$ an equivalence relation on $X$, then $X/\sim$ is compact. Similarly, if $X$ is connected, $X/\sim$ is connected.

11. Let $X$ be connected, and $Y$ discrete. Show that every continuous function $f : X \to Y$ is constant; that is, there is an element $c \in Y$ with $f(x) = c$ for every $x \in X$. 