1. Finish the proof from class: show that the function $\mathcal{F} : (I^2 / \sim) \to T^2_{a,b}$ given by

$$\mathcal{F}([[(x,y)]) = (a + b \cos(u))(\cos(v)i + \sin(v)j) + b \sin(u)k$$

is a homeomorphism, where $u = 2\pi x - \pi$, and $v = 2\pi y$.

2. Show that the planar model for $S^2$ is correct. That is, let $D^2$ be the closed unit disk in the plane, and define an equivalence relation on it by declaring $(x,y) \sim (-x,y)$ if $(x,y)$ is on the boundary circle (that is, $x^2 + y^2 = 1$). Show that the function $g : D^2 \to S^2$ given by

$$g(x,y) = \sqrt{1-y^2} \left( \cos \left( \frac{\pi x}{\sqrt{1-y^2}} \right) i + \sin \left( \frac{\pi x}{\sqrt{1-y^2}} \right) j \right) + yk$$

is constant on equivalence classes under $\sim$, and induces a homeomorphism $\overline{g} : (D^2 / \sim) \to S^2$.

3. Use cut-and-paste techniques to determine what surfaces the following words describe:

(a) $abcc^{-1}ab^{-1}$
(b) $abccab^{-1}$
(c) $abcc^{-1}a^{-1}b$
(d) $abcc^{-1}b$
(e) $abcc^{-1}a^{-1}b^{-1}$
(f) $abcc^{-1}b^{-1}$