Recall that the geodesic curvature of a curve $\gamma$ on a surface $X$ is given by the function

$$\kappa_g(s) = \gamma''(s) \cdot (n \times \gamma'(s))$$

where $n$ is the unit normal to $X$ at $\gamma(s)$.

1. Let $X$ be the $xy$-plane in $\mathbb{R}^3$. Compute the geodesic curvatures of the following curves:
   (a) The straight line: $\gamma(t) = (at, bt, 0)$.
   (b) The circle of radius $a$: $\gamma(t) = (a \sin(t), a \cos(t), 0)$.

2. Let $X$ be the sphere in $\mathbb{R}^3$ of radius $a$, with parameterisation

   $$r(u, v) = (a \sin(u) \sin(v), a \cos(u) \sin(v), a \cos(v)).$$

   (a) Compute the geodesic curvature of the latitudinal circle $v = v_0$ (that is, $v$ is constant).
   (b) Conclude that the equator is the only latitudinal circle which is a geodesic.
   (c) Why does your computation of the geodesic curvature of the equator disagree with your answer from 1.(b)?

3. Show that if $f : X \to Y$ is an isometry, then $f \circ \gamma$ is a geodesic on $Y$ if and only if $\gamma$ is a geodesic on $X$.

4. Use the previous two results to show that every great circle on the sphere is a geodesic.

5. Let $X$ be the torus obtained by revolving the circle of radius $b$ centered at distance $a$ from the origin around the $z$ axis. $X$ has a parameterisation

   $$r(u, v) := (a + b \cos u)(\cos v \mathbf{i} + \sin v \mathbf{j}) + b \sin u \mathbf{k}$$

   You showed in previous practice problems that the unit normal to $X$ is given (up to sign) by

   $$n = \cos u(\cos v \mathbf{i} + \sin v \mathbf{j}) + \sin u \mathbf{k}.$$

   (a) Compute the geodesic curvature of the “horizontal” circle $u = u_0$. For what values of $u_0$ is it a geodesic?
   (b) Compute the geodesic curvature of the “vertical” circle $v = v_0$. For what values of $v_0$ is it a geodesic?