1. Compute $H^*(\mathbb{R}P^3 \times \mathbb{C}P^5; k)$ (as a ring) for $k = \mathbb{Z}$ and $\mathbb{Z}/2$.

2. Compute $H^*(\mathbb{R}P^3 \times \mathbb{R}P^2; \mathbb{Z}/2)$ (as a ring).

3. Let $X, Y$ be connected CW complexes with finitely many cells in each dimension.
   (a) Compute $H^*(X \vee Y)$ as a ring, in terms of the rings $H^*(X), H^*(Y)$.
   (b) Show that there is an isomorphism of graded abelian groups
   $$H^*(X \times S^n) \cong H^*(X \vee \Sigma^n X),$$
   where $\Sigma^n X$ is the iterated suspension.
   (c) Show that unless $X$ has the homology of a point, this cannot be an isomorphism of rings. Conclude that there cannot exist a homotopy equivalence
   $$X \times S^n \simeq X \vee \Sigma^n X.$$

4. Let $f : X \to Y$ be a continuous map, $a \in H_p(X), \phi \in H^q(Y)$. Prove that
   $$f_*(f^*(\phi) \cap a) = \phi \cap f_*(a).$$
   **Hint:** Write out both sides of the equation on the chain level.

5. Let $M$ denote a compact, oriented $n$-manifold. Let $f : S^n \to M$ be a continuous map of degree $d$; i.e.,
   $$f_*[S^n] = d[M].$$
   Use the results of the previous problem and Poincaré duality to show: for $0 < q < n$,
   show that every $x \in H_q(M)$ satisfies $d \cdot x = 0$.

6. Use the results of the previous problem to:
   (a) describe all possible degrees of maps $S^n \to T^n$ (the $n$-torus).
   (b) give a new proof of the fact that every element of $H_q(\mathbb{R}P^{2m+1})$ is 2-torsion, for $0 < q < 2m + 1$. 
