3. Solve graphically the following non-standard linear programming problem: Draw a graph with feasible region clearly marked and with all its corner points calculated. Write down all basic feasible solutions, and write down the values of \((x, y)\) for which \(F\) takes its maximum value.

Maximise \( F = 51x + 34y \)

subject to

- \(10x + 15y \leq 120\)
- \(9x + 6y \leq 63\)
- \(13x + 13y \geq 65\)

with \(x \geq 0\) and \(y \geq 0\).

Answer: The correct graph, list of corner points, all basic feasible solutions, and

\(F = 357\) is the maximum value of

The objective function occurring at

\((x, y) = (t, 10t - \frac{3}{2}t^2)\) for \(3 \leq t \leq 7\)
3. Solve graphically the following non-standard linear programming problem: Draw a graph with the feasible region clearly marked, each of the boundary lines clearly marked and with all the corner points calculated. Include on your graph at least three 'constant objective function' straight lines, including one that goes through the points in the feasible region that give the maximum value of $F$. Using the graph of the feasible region and the three lines drawn give an argument for which values of $(x, y)$ give the maximum value of $F$ in the feasible region. Do not use the values of the objective function at ALL the corner points in your argument. You may use the values of the objective function at some corner points in your argument.

Maximise $F = 10x + 10y$

subject to $9x + 12y \geq 36$
$x \leq 9$
$4x + 3y \leq 48$
$y \leq 8$

with $x \geq 0$ and $y \geq 0$. 
Solution: 

We can rewrite the problem as

Maximise \( F = 10x + 10y \)

Subject to \( 3x + 4y \geq 12 \)
\( x \leq 9 \)
\( y \leq 8 \)
\( 4x + 3y \leq 48 \)

with \( x \geq 0 \) and \( y \geq 0 \).

The boundary lines are

\( 3x + 4y = 12 \)
\( 4x + 3y = 48 \)
\( x = 9 \)
\( y = 8 \)
\( x = 0 \)
\( y = 0 \)
are 'corner points'

They are (0,3), (4,0), (9,0), (9,4), (6,8) and (0,8).
The value of $F$ at the corner point $(9,0)$ is $F = 90$.

The value of $F$ at the corner point $(9,4)$ is $F = 130$.

The value of $F$ at the corner point $(6,8)$ is $F = 140$.

At the point $(9,8)$ which is outside the feasible region, $F = 170$.

We have drawn the three constant objective function straight lines:

- $F = 90 = 10x + 10y$
- $F = 140 = 10x + 10y$
- $F = 170 = 10x + 10y$

See diagram on previous page.
Clearly as $F$ increases the graph of the objective function moves to the upper right. The graph of $F=140$ cuts through the feasible region at only one point, the corner point $(6, 8)$.

Any value of $F$ larger than $F=140$ will lead to a straight line to the upper right of the line produced from $F=140$ and hence must lie wholly outside the feasible region. Any other objective function with points lying in the feasible region must be to the lower left of the line given by $F=140$ and hence must be associated with a value less than 140. We hence conclude that $F=140$ is the maximum that $F$ can attain in the feasible region.
Hence the solution is

\[ F = 140 \text{ at } (x, y) = (6, 8) \]