Use the simplex method to solve the following standard maximum problem.

Maximise \[ g = 12y_2 + 13y_3 \]

subject to \[ \begin{align*}
2y_1 + 2y_2 & \leq 6 \\
12y_1 + 6y_2 + 12y_3 & \leq 24 \\
15y_1 & - 16y_3 \leq 8
\end{align*} \]

with \( y_1 \geq 0, y_2 \geq 0 \) and \( y_3 \geq 0 \).

At each step show clearly the row operation(s) that you perform and clearly circle the pivot element. Inspect your final tableau and state the maximum possible value of \( g \) and all the values of \( (y_1, y_2, y_3) \) for which this maximum occurs.
We introduce slack variables $s_1, s_2, s_3$ so that the problem constraints become

$$2y_1 + 2y_2 + s_1 = 6,$$
$$12y_1 + 6y_2 + 12y_3 + s_2 = 24,$$

and $$15y_1 - 16y_3 + s_3 = 8,$$

where $s_1 \geq 0$, $s_2 \geq 0$, and $s_3 \geq 0$.

We rewrite the system as a Simplex Tableaux

$$\begin{array}{cccccccc}
y_1 & y_2 & y_3 & s_1 & s_2 & s_3 & p & \text{RHS} \\
2 & 2 & 0 & 1 & 0 & 0 & 0 & 6 \\
12 & 6 & 0 & 1 & 0 & 0 & 0 & 24 \\
15 & 0 & -16 & 0 & 0 & 1 & 0 & 8 \\
0 & -12 & -13 & 0 & 0 & 0 & 1 & 0 \\
\end{array}$$

\text{Quotients}

$2 = \frac{24}{12}$

\text{pivot column}

$\frac{1}{12} R_2 \rightarrow R_2$

Pivot element is circled.
\[
\begin{bmatrix}
2 & 2 & 0 & 1 & 0 & 0 & 0 & 6 \\
1 & \frac{1}{2} & 1 & 0 & \frac{1}{12} & 0 & 0 & 2 \\
15 & 0 & -16 & 0 & 0 & 1 & 0 \\
0 & -12 & -13 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]

16\, R_2 + R_3 \rightarrow R_3

12\, R_2 + R_4 \rightarrow R_4

\[
\begin{bmatrix}
2 & 0 & 1 & 0 & 0 & 0 & 0 & 6 \\
1 & \frac{1}{2} & 1 & 0 & \frac{1}{12} & 0 & 0 & 2 \\
3 & 0 & 0 & 0 & \frac{4}{3} & 1 & 0 & 40 \\
13 & -\frac{11}{2} & 0 & 0 & \frac{13}{12} & 0 & 1 & 26
\end{bmatrix}
\]

- Quotients
  - \frac{6}{2} = 3
  - \frac{3}{12} = \frac{1}{4}
  - \frac{9}{8} = \frac{9}{8}

\frac{1}{2}\, R_1 \rightarrow R_1

\[
\begin{bmatrix}
1 & 1 & 0 & \frac{1}{2} & 0 & 0 & 0 & 3 \\
1 & \frac{1}{2} & 1 & 0 & \frac{1}{12} & 0 & 0 & 2 \\
3 & 8 & 0 & 0 & \frac{4}{3} & 1 & 0 & 40 \\
13 & -\frac{11}{2} & 0 & 0 & \frac{13}{12} & 0 & 1 & 26
\end{bmatrix}
\]

- \frac{1}{2}\, R_1 + R_2 \rightarrow R_2
- 8\, R_1 + R_2 \rightarrow R_3
- \frac{11}{2}\, R_1 + R_4 \rightarrow R_4

pivot column
\[
\begin{bmatrix}
1 & 1 & 0 & \frac{1}{2} & 0 & 0 & 0 & 3 \\
\frac{1}{2} & 0 & 1 & -\frac{1}{4} & \frac{1}{12} & 0 & 0 & \frac{1}{2} \\
23 & 0 & 0 & -4 & \frac{4}{3} & 1 & 0 & 16 \\
\frac{37}{2} & 0 & 0 & \frac{11}{4} & \frac{13}{12} & 0 & 1 & \frac{85}{2}
\end{bmatrix}
\]

STOP since there are no negative elements in bottom row.

We deduce that the maximum value of \( g \) is \( \frac{85}{2} \) occurring at \((y_1, y_2, y_3) = (0, 3, \frac{1}{2})\).

The slack variables at this point have values \((s_1, s_2, s_3) = (0, 0, 16)\).
Answer check: (We note that \( y_1, y_2, y_3 \geq 0 \) at \((y_1, y_2, y_3) = (0, 3, \frac{1}{2})\).)

If we label

\[ 2y_1 + 2y_2 \leq 6 \quad (i) \]
\[ 12y_1 + 6y_2 + 12y_3 \leq 24 \quad (ii) \]
\[ 15y_1 + 16y_3 \leq 8 \quad (iii) \]

We note that at \((y_1, y_2, y_3) = (0, 3, \frac{1}{2})\)

LHS of \((i)\) \(= 2 \times 0 + 2 \times 3 = 6 \leq 6 = \text{RHS of } (i) \)
(and \( s_1 = 0 \)) so \text{OK}.

LHS of \((iii)\) \(= 12 \times 0 + 6 \times 3 + 12 \times \frac{1}{2} \)
\[= 18 + 6 = 24 \leq 24 = \text{RHS of } (ii) \]
(and \( s_2 = 0 \)) so \text{OK}

LHS of \((ii)\) \(= 15 \times 0 - 16 \times \frac{1}{2} = -8 \leq 8 = \text{RHS of } (iii) \)

So \text{OK.} (and \( s_3 = 8 - (-8) = 16 \)).

Now \( g = 12y_2 + 13y_3 = 12 \times 3 + 13 \times \frac{1}{2} \)
\[= 36 + \frac{13}{2} = \frac{72 + 13}{2} = \frac{85}{2} \]
so \text{OK.}