5. (a) Find the slope of the curve

\[ x^3y^2 + e^{x(y-2)} = 1 + \frac{1}{e} \]

at the point \((1,1)\)

(b) Two variables are related by the equation

\[ xy^2 - x^3y = 2 \]

When \(x = 1\) and \(y = 2\), \(x\) is increasing at a rate of 4 units a day. What is the rate of change of \(y\) when \(x = 1\) and \(y = 2\)?

9. You are walking in a straight line, at a constant rate, towards your local pub, which has a very loud band playing. When you are 100 metres from the pub a sound intensity measuring device you just happen to be carrying, and you have pointing towards the pub, measures the sound intensity at 5 units. When you are 50 metres from the pub the measured instantaneous rate of increase of intensity is 2 units per second.

Assuming that the sound intensity at any point is inversely proportional to the square of the distance from the point source (and assuming that the band in the pub can be modelled as a point source) at what speed (in metres per second) are you walking?
5(a) \[ x^3 y^2 + e^{x(y-2)} = 1 + \frac{1}{e} \quad (\ast) \]

Differentiating both sides of \((\ast)\) with respect to \(x\) gives

\[
\frac{d}{dx}(x^3 y^2) + \frac{d}{dx}(e^{x(y-2)}) = \frac{d}{dx}(1 + \frac{1}{e})
\]

\[
\Rightarrow 3x^2 y^2 + x^3 \frac{dy}{dx} + \left(\frac{d}{dx}(x(y-2))\right) e^{x(y-2)} = 0
\]

\[
\Rightarrow 3x^2 y^2 + 2x^3 y \frac{dy}{dx} + \left[(y-2) + x \frac{dy}{dx}\right] e^{x(y-2)} = 0
\]

Now solving for \(\frac{dy}{dx}\) gives

\[
(2x^3 y + x e^{x(y-2)}) \frac{dy}{dx} = -3x^2 y^2 - (y-2) e^{x(y-2)}
\]
So \( \frac{dy}{dx} = \frac{-3x^2y^2 - (y-2)e^{x(y-2)}}{2x^3y + xe^{x(y-2)}} \)

At \( x=1, y=1 \)

\[ \frac{dy}{dx}\bigg|_{x=1, y=1} = \frac{-3 - (1-2)e^{(1-2)}}{2 + e^{(1-2)}} = \frac{-3 + e^{-1}}{2 + e^{-1}} \]
(b) \[ x y^2 - x^3 y = 2 \] (*)

Differentiating both sides of (*) with respect to \( x \) gives

\[
\frac{d}{dx} (x y^2) - \frac{d}{dx} (x^3 y) = \frac{d}{dx} (2)
\]

\[
\Rightarrow y^2 + x \frac{dy}{dx} - 3x^2 y - x^3 \frac{dy}{dx} = 0
\]

\[
\Rightarrow y^2 + 2xy \frac{dy}{dx} - 3x^2 y - x^3 \frac{dy}{dx} = 0
\]

So

\[
(2xy - x^3) \frac{dy}{dx} = 3x^2 y - y^2
\]

\[
\Rightarrow \frac{dy}{dx} = \frac{3x^2 y - y^2}{2xy - x^3}
\]
At $x=1, y=2$

\[ \frac{dy}{dx} \bigg|_{x=1 \atop y=2} = \frac{3 \cdot 1^2 \cdot 2 - 2^2}{2 \cdot 1^2 \cdot 2 - 1^3} \]

\[ = \frac{6 - 4}{4 - 1} = \frac{2}{3} \]

Now using the chain rule gives

\[ \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} \]

We won't \( \frac{dy}{dt} \bigg|_{x=1 \atop y=2} \)

So

\[ \frac{dy}{dt} \bigg|_{x=1 \atop y=2} = \frac{dy}{dx} \bigg|_{x=1 \atop y=2} \cdot \frac{dx}{dt} \bigg|_{x=1 \atop y=2} \]

We are told that \( \frac{dx}{dt} \bigg|_{x=1 \atop y=2} = 4 \) so
\[
\frac{dy}{dx} \bigg|_{x=1 \atop y=2} = \frac{2}{3} \cdot 4 = \frac{8}{3}
\]

The answer is \( \frac{8}{3} \) units a day.
Let the distance from me to the pub be \( x \) metres at time \( t \) seconds. Let the intensity of sound be \( I \).

We are told that

\[
I \propto \frac{1}{x^2}
\]

So we define a constant \( C \) so that

\[
I = \frac{C}{x^2} \quad (\alpha)
\]

We are told that

\[
I = S \quad \text{when} \quad x = 100
\]

So

\[
S = \frac{C}{100^2} \quad \text{giving} \quad C = 50,000
\]
Hence \[ I = \frac{50,000}{x^2} \] (\(\beta\))

We want \( \frac{dx}{dt} \) which is constant, so we use the chain rule to give

\[
\frac{dI}{dt} = \frac{dI}{dx} \cdot \frac{dx}{dt}
\]

Using (\(\beta\)) gives

\[
\frac{dI}{dx} = -2 \times 50,000 \frac{1}{x^3}
\]

\[
= -\frac{100,000}{x^3}
\]

We are told that

\[
\frac{dI}{dt} = 2 \text{ at } x = 50
\]

So

\[
\left. \frac{dI}{dt} \right|_{x=50} = \left. \frac{dI}{dx} \right|_{x=50} \cdot \left. \frac{dx}{dt} \right|_{x=50}
\]
Hence

\[ 2 = -\frac{100,000}{50^3} \cdot \frac{dx}{dt} \]

so

\[ \frac{dx}{dt} = -\frac{2 \times 125,000}{100,000} \]

\[ = -2.5 \quad \text{at} \quad x=50 \]

Since \( \frac{dx}{dt} \) is constant, \( \frac{dx}{dt} = -2.5 \) at any time (or distance).

Hence, I am walking at 2.5 metres per second towards the band.