1. The concentration, \( x \) g/cm\(^3\), of a drug in a body organ at time \( t \) sec. after it was given to the patient is given by the formula \( x = 0.2(1 - e^{-\frac{t}{40}}) \).

Nurses need to use a simpler formula which still gives an accurate value for the concentration.

(a) Use a Taylor series expansion about \( t = 0 \) to obtain a formula up to a cubic term in \( t \) which approximates the above formula.
(b) Use your formula to calculate the concentration of the drug when \( t = 20 \) seconds.
(c) Calculate the exact concentration as given by the exponential formula and show that the percentage error is less than 1%.

2. The equation, \( y = \frac{b}{2}(e^{ax} + e^{-ax}) \) represents a cable hanging between two vertical poles standing on horizontal ground, e.g., an electricity cable.

(a) If for a particular cable, \( a = 0.018, b = 8 \), and the lowest point on the cable is situated on the y-axis, calculate the coordinates of the lowest point.
(b) The amount of "sag", i.e., the difference in vertical height between the lowest point on the cable and its point of support, is of importance to linesmen (maintenance men). Calculate the sag in a cable if the support poles are 40 metres apart.
(c) A linesman would like to have a simpler formula to calculate the sag. On behalf of the linesman, determine a Taylor series for \( x \) (up to the term in \( x^3 \)) about \( x = 0 \) which approximates the equation for the cable.
(d) Use this formula to calculate the sag between poles 40 m apart and compare your answer with that obtained by using the exponential formula.

3. For the initial value problem \( \frac{dy}{dx} = x + y^2, \quad y(1) = 1 \). Calculate \( y'(1), y''(1) \) and \( y'''(1) \) and write a polynomial of degree three in \( x-1 \) which approximates the solution of the initial value problem when \( x = 1 \).