Using the hint, we let
\[ x_1 = \text{amount invested with Grabbit} \]
\[ x_2 = \text{amount invested with Gravitt} \]

Clearly, we must have \( x_1, x_2 \geq 0 \).

Since the student has at most \$21,000 to invest, we must have:
\[ x_1 + x_2 \leq 21,000 \]

Since Gravitt will only invest up to \$15,000, we need:
\[ x_2 \leq 15,000 \]

In order to not invest more than twice the amount with Gravitt: as with Grabbit, we require
\[ x_2 \leq 2x_1 \]

We want to maximise the interest, say \( I \), which is given by:
\[ I = 0.04x_1 + 0.06x_2 \]

So the linear programming problem is:

Maximise \( I = 0.04x_1 + 0.06x_2 \)

Subject to
\[ x_1 + x_2 \leq 21,000 \]
\[ x_2 \leq 15,000 \]
\[ -2x_1 + x_2 \leq 0 \]
\[ x_1, x_2 \geq 0 \]
(a) Solving this graphically, we sketch the feasible region:

Since the optimal value must occur at a corner point of this (banded) feasible region, we evaluate $I$ at all corner points.

<table>
<thead>
<tr>
<th>corner point $(x_1, x_2)$</th>
<th>$I = 0.04x_1 + 0.06x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>0</td>
</tr>
<tr>
<td>(21000, 0)</td>
<td>$0.04 \times 21000 = 840$</td>
</tr>
</tbody>
</table>
| (7000, 14000)            | $0.04 \times 7000 + 0.06 \times 14000 = 1120$ | Max

So the maximum possible interest is $1120$, when $7000$ is invested with Grabbit and $14000$ is invested with Growitt.

Check in constraints:
- $7000 + 14000 \leq 21000$ ✔
- $14000 \leq 15000$ ✔
- $7000, 14000 \geq 0$ ✔
- $-2(7000) + 14000 = 0 \leq 0$ ✔

all OK.
(b). Using the simplex method, we first rewrite the problem with slack variables:

\[
\begin{align*}
& x_1 + x_2 + s_1 = 21000 \\
& x_2 + s_2 = 15000 \\
& -2x_1 + x_2 + s_3 = 0 \\
& -0.04x_1 - 0.06x_2 + I = 0
\end{align*}
\]

Then the initial simplex tableau is:

\[
\begin{bmatrix}
BV & x_1 & x_2 & s_1 & s_2 & s_3 & I & RHS \\
\hline
s_1 & 1 & 1 & 0 & 0 & 0 & 0 & 21000 \\
s_2 & 0 & 0 & 1 & 0 & 0 & 0 & 15000 \\
s_3 & -2 & 1 & 0 & 0 & 1 & 0 & 0 \\
I & -0.04 & -0.06 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

↑

Ratio test:

- \( \frac{21000}{1} \) smaller ratio
- \( \frac{15000}{1} \)
- \( \frac{0}{1} \)

\( \frac{21000}{7000} = 3 \), \( \frac{15000}{7500} = 2 \)

Row operations:

- \( R_1 - 3R_3 \rightarrow R_1 \)
- \( R_2 - 2R_3 \rightarrow R_2 \)
- \( R_3 + 2R_1 \rightarrow R_3 \)
- \( R_4 + 0.06R_3 \rightarrow R_4 \)

Since there are no more negatives in the bottom row, we are done.

We read off: Max value for I is 1120, when

\[
\begin{align*}
x_1 = 7000, & \quad s_1 = 1000, \\
x_2 = 14000, & \quad s_3 = 0
\end{align*}
\]

In summary: Invest $7000 with Grabbit and $14000 with Growitt for a maximum interest of $1120. (as in (a)).