LECTURE II:
Linear Programming:
Simplex Method.

1: The Simplex Algorithm.
2: Reading off a solution
3: One solution, no solution
   or infinitely many?
   how can we tell?

Example 2.
Maximize \( P = 4x_1 + 5x_2 + 2x_3 \)
subj to
\[
2x_1 + 3x_2 + x_3 \leq 900 \\
5x_1 + 2x_2 + 2x_3 \leq 1200 \\
x_1, x_2, x_3 \geq 0.
\]

Solution:
Step 1: Add slack variables

Step 2: Rewrite objective function

Step 3: Set up Simplex Tableau

<table>
<thead>
<tr>
<th>BV</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( P )</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Step 4: Apply the Greedy Rule
to choose entering variable

Step 5: Use Ratio Test to choose
exiting variable.

Step 6: Perform pivot operation
to update tableau.

<table>
<thead>
<tr>
<th>BV</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( P )</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Step 7: Are there negative values
in the bottom row?
If No \( \to \) you are finished
If Yes \( \to \) return to Step 4.
Step: Are there negative values in the bottom row?

No more negative entries so we are finished.

Solution:

*Check your solution.

Try this one yourself!

Maximize $P = 6x_1 + 4x_2$

subject to

$\begin{align*}
2x_1 + 3x_2 & \leq 240 \\
4x_1 + 2x_2 & \leq 180 \\
x_1, x_2 & \geq 0
\end{align*}$

Rewrite constraints.

So solution is

Check your solutions!
Example

Max \[ P = x_1 + 2x_2 \]

subj to \[ x_1 - x_2 \leq 1 \]
\[ -2x_1 + x_2 \leq 4 \]
\[ x_1, x_2 \geq 0 \]

Rewrite:

Another Example

max \[ P = 4x_1 + 8x_2 \]

subj to \[ 2x_1 + 4x_2 \leq 200 \]
\[ 6x_1 + x_2 \leq 100 \]
\[ x_1, x_2 \geq 0 \]

Rewrite:

It tells us that there is

The solution we can read off is

The bottom row says