LECTURE 15:
DERIVATIVES

1. Definition of a derivative
2. Basic Rules
3. Product Rule
   Quotient Rule
   Chain Rule
4. Examples.

The derivative of a function is also a function (not a number): we write it as:
(If \( y = f(x) \))

\[ f'(x), \frac{dy}{dx}, \frac{df}{dx} \]

which all mean the same thing

... the function obtained by differentiating \( f \) at all points \( x \) in its domain.

2. Calculating derivatives from first principles.

Example: Let \( f(x) = \frac{1}{x} \).
Find \( f'(x) \).

\[ f'(x) \]
4. Examples.

1. \( f(x) = e^{3x} \)

2. \( f(x) = 3x^2 + \log_e(5x) \)

3. \( f(x) = x^2 \log_e(x) \).

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The Chain Rule.

Let \( y = (f \circ g)(x) \).

\[ = f(g(x)) \]

Put \( u = g(x) \).

Then \( y = f(u) \).

We have two ways of writing the Chain Rule:

1. \( (f \circ g)'(x) \)

   \[ = f''(g(x)) \cdot g'(x) \]

2. \[ \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \]

Either version is OK.

Examples.

1. Let \( y = (x^4 + 5x^6)^5 \).

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*The answer must be an explicit function of \( x \).*

2. Let \( f(x) = \sqrt{x^4 + 3x^2} \).
3: Let \( y = \log_e (4x^3 + 7x^2 + 2x + 1) \)

Now try these:

1. \( y = \sin(x) \cdot (3x^2 + 2x) \)

2. \( y = \frac{x^3 + 2x}{\cos(x)} \)

3. (i) \( y = \sin(x^2) \)
   (ii) \( y = \sin^2(x) \)

4. \( y = \log_e (\sin(x^3 + 2)) \)