LECTURE 19: Introduction to Differential Equations.

1. What is a DE?
2. Solving DE’s.
4. Slope fields
5. Families of solutions.

Introduction to Differential Equations. BZ pp. 948-949.

Last week we looked at functions and found their derivatives (explicit & implicit).

Conversely, given \( \frac{dy}{dx} \), can we find \( y \) as a function of \( x \)?

A first order differential equation is an equation for an unknown function \( f(x) \) which also involves the function’s derivative.

Examples of first order DE’s.

(a) \( \frac{dy}{dx} = 3xy \) 

(b) \( x^2 \frac{dy}{dx} = y^2 + y + 2 \) 

(c) \( \frac{dy}{dx} + 2xy = x^2 \) 

(d) \( \frac{dy}{dx} = \frac{-x}{y} \) 

(e) \( \frac{dy}{dx} = 3x^2 \)

Solving DE’s: Finding \( f(x) \) explicitly:

A solution of a DE is an explicit expression for the unknown function \( f(x) \) which satisfies the original equation.

Example 1:

\[ \frac{dy}{dx} = 2x \cdot y \]

One solution to this DE is \( y = e^{x^2} \).

Why?
Now consider \( y = e^{(x^2+8)} \).

Example 2:

\[
\frac{dy}{dx} = x - y.
\]

Verify that \( y = x - 1 + 3e^{-x} \) is a solution to the differential equation.

Solution Curves / Numerical Solutions.

\[
\frac{dy}{dx} = x - y
\]

has solution curve:

Picture of solution.

Sometimes we have to rely on a numerical solution.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>0.1</td>
<td>1.8145</td>
</tr>
<tr>
<td>0.2</td>
<td>1.6562</td>
</tr>
<tr>
<td>0.3</td>
<td>1.5225</td>
</tr>
</tbody>
</table>
Solution Family

general solution.

\[ \frac{dy}{dx} = x - y. \]

Some solutions are:

Check.

We refer to the solution

\[ y = x - 1 + Ae^{-x}, \quad A \in \mathbb{R}, \]

as the

of the DE.

The curves \( y = x - 1 + Ae^{-x}, A \in \mathbb{R} \), constitute the solution family of curves (there are an infinite number).

We may also have solutions to the DE called

These are solutions which differ from the general form.

Example:

\[ y = Cx - \frac{1}{4} C^2 \]

is a solution family for

\[ y = x \frac{dy}{dx} + \frac{1}{4} \left( \frac{dy}{dx} \right)^2 \]

but so is \( y = x^2 \).

Constructing a DE from an implicit function.

Example:

Find a DE of which

\[ 3y^2 + 2x^3 y + x = 4 \quad (*) \]

is one (implicit) solution.

Solution

Differentiate both sides of (*), with respect to \( x \).