3 Linear Equations

1. Types of solutions
   - geometric interpretation.

2. Overdetermined systems
   - Underdetermined systems

3. Examples.

We can't always obtain the row reduced form that we require. Why?

Example: 2 equations
   2 unknowns.

3 types of solutions:

We have a parametric solution.

is a solution for any value of t.

This classification of solution types extends to any system where # variable = # equations .... but there are more possibilities.

So far we have seen examples which have the same number of equations as variables. However this may not always be the case.

Overdetermined systems.
# equations > # variables.
All solution types are possible.

Underdetermined systems.
# equations < # variables.
Intersection cannot be a point - but other solution types possible.
Overdetermined Systems.

Solve: \[ 2x - y = 3 \]
\[ x - y = -2 \]
\[ 3x + 2y = 6 \]

What does this look like geometrically?

There is no common point of intersection.

For 3 equations in 2 variables (3 straight lines in the plane), there are many possible solutions:

(a) one solution
(b) no solution

(i) consistent and independent

(ii) no solution
(iii) no solution

Other possibilities:
- two equations represent the same line (so we are back to the two line case).
- all 3 equations represent the same line \( \rightarrow \) infinitely many solutions.

How do we recognize these solution types from the matrices?
Underdetermined Systems.

less equations than variable

Example:

\[ x_1 + 2x_2 + 2x_3 = 4 \]
\[ 4x_1 - 6x_2 + x_3 = 2 \]

Is it possible to obtain a unique solution?

We choose the variable which does not appear as a row leader, to be the parameter.

Geometrically, this means that two planes meet in a line.
Each value of \( t \), will give a different point on the line.

eg.
Extra notes:

1. If a row of zeroes $[0 \ 0 \ \ldots \ 0 \ 1 \ 0]$ appears anywhere in your calculations—shift it to the bottom, it is of no further use. (But don't omit it).

2. If a row of the form $[0 \ 0 \ \ldots \ 0 \ 1 \ a]$ a ≠ 0 appears at any stage, stop! This tells you that there are no solutions.

Examples:

1. \[ \begin{align*}
     x - 3y &= 2 \\
     2x - 5y - 2z &= 7 \\
     -x + y + 4z &= -3
\end{align*} \]

2. \[ \begin{align*}
     x - 3y &= 2 \\
     2x - 5y - 2z &= 7 \\
     -x + y + 4z &= -8
\end{align*} \]
Summary of Method.

* Use allowable row operations to obtain "1"s in diagonal entries (that is, $a_{ii}$).
* Use allowable row operations to obtain "0"s elsewhere in columns containing row leaders.
* Zero rows should be moved to the bottom.
* Check for inconsistencies: $[0 \ 0 \ldots \ 0 \ 1 \ a] \ a_{to} \Rightarrow$ no solution.
* Read off solutions: If the number of row leaders = number of variables, then there is a unique solution.
* Otherwise - infinitely many solutions (parametric).