LECTURE 33.

Logistic Model with Harvesting.

LOGISTIC MODEL WITH HARVESTING.

* From our knowledge of quadratic equations, we know that if there will be two equilibrium solutions. If there will be no equilibrium solutions.

Example

\[
\frac{dx}{dt} = 5x - x^2 - 6 \quad x(0) = x_0
\]

This model may be a population of x thousand birds in a park who are culled at a rate of 6,000/year. Find and classify the equilibrium points.

Draw a phase line.

General solution

\[
\frac{dx}{dt} = -(x-3)(x-2) \quad \text{separable equation.}
\]
Now \( \pi(0) = \pi_0 = 80 \),
\[ \pi(0) \]

So \( \pi(t) = \)

Can the population die out?
That is, can \( \pi = 0 \) for some \( T \)?

This answer clearly depends on \( \pi_0 \).

Death of a population

We have 3 cases to consider:

(i) If \( \pi_0 > 3 \) then
\[ 3 - \pi_0 < 0 \] and \[ 2 - \pi_0 < 0 \]

(ii) If \( 2 < \pi_0 < 3 \),
Then \( 3 - \pi_0 > 0 \) and \( 2 - \pi_0 < 0 \)

(iii) If \( \pi_0 < 2 \) then
\[ 3 - \pi_0 > 0 \] and \[ 2 - \pi_0 > 0 \]
hence population death can occur if \( r < 2 \), at

Linearizing \( u' \) about an equilibrium point.

Suppose that for a \( \xi_0 \),
\[
\frac{dx}{dt} = G(x)
\]
we have \( G(x_0) = 0 \) for some \( x_0 \).
So \( x_0 \) is an equilibrium solution of the above DE.
(i.e. can approximate \( G(x) \) near \( x_0 \) by substituting \( x = x_0 + \xi \).

So our DE becomes
\[
\frac{du}{dt} \approx bu
\]
What about stability?
\[
u \approx u_0 e^{kt}
\]
So \( x \approx x_0 + u_0 e^{kt} \)
What happens as \( t \) becomes large?
It all depends on \( b \).

So \[
\frac{du}{dt}
\]

We approximate \( G(u + \xi) \) by its Taylor expansion about \( x = x_0 \). - we only need the linear term, the higher order terms are very small.

Example
Let's go back to
\[
\frac{dx}{dt} = 5x - x^2 - 6 \quad x(0) = 2.
\]
suppose that \( x_0 \) is close to the equilibrium value \( x = 2 \).
We make the substitution \( x = u + 2 \) with \( |u| < 1 \) (small)
The DE becomes
Since \( z_0 \) is close to \( z=2 \),
\( U_0 \) is close to zero.
\( \therefore U_0^2 \) is very small.
So we disregard the \( U_0^2 \) term and leave the linear term.
\[
\frac{du}{dt} \approx u \quad \text{linearized around } u=0.
\]

The other equilibrium.

Suppose \( x_0 \) is near \( z=3 \).
Substitute \( x = v + 3 \) with \(|v| \ll 1\).

So \[
\frac{dv}{dt} \approx -v, \quad v(0) = v_0.
\]