Out of 8 - Parts (a) and (b) each out of 4.

(a) The feasible region is the same for parts (i) and (ii) and is given by the inequalities:

\[
\begin{align*}
3x - y &\leq 24 \\
x + y &\leq 16 \\
5x + 4y &\geq 40 \\
-x + 2y &\leq 20 \\
x &\geq 0, y &\geq 0
\end{align*}
\]

Consider the bounding lines:

1. \(3x - y = 24\) When \(x = 0\), \(y = -24\). When \(y = 0\), \(x = 8\).
2. \(x + y = 16\) When \(x = 0\), \(y = 16\). When \(y = 0\), \(x = 16\).
3. \(5x + 4y = 40\) When \(x = 0\), \(y = 10\). When \(y = 0\), \(x = 8\).
4. \(-x + 2y = 20\) When \(x = 0\), \(y = 10\). When \(y = 0\), \(x = -20\).

After testing whether the origin lies in each half-plane, we find that the feasible region is the shaded region below:

We need to determine the two corner points not on the axes.
Solving simultaneously, for the intersection of lines (1) and (2) we have:

\[
(1) + (2) : \quad 4x = 40 \quad \Rightarrow \quad x = 10 \quad \Rightarrow \quad y = 16 - x = 16 - 10 = 6, \quad \text{i.e.} \ (10, 6).
\]

And for the intersection of lines (2) and (4) we have:

\[
(2) + (4) : \quad 3y = 36 \quad \Rightarrow \quad y = 12 \quad \Rightarrow \quad x = 16 - y = 16 - 12 = 4, \quad \text{i.e.} \ (4, 12).
\]
(i) We want to maximise $P = 2x + y$ on the above feasible region. The optimal value of $P$ will occur at a corner point. We have

\[
\begin{align*}
P(8, 0) &= 2(8) + 0 = 16 \\
P(0, 10) &= 2(0) + 10 = 10 \\
P(10, 6) &= 2(10) + 6 = 26 \\
P(4, 12) &= 2(4) + 12 = 20 \\
\end{align*}
\]

So the optimal solution is $(x, y) = (10, 6)$ giving the optimal value $P = 26$.

CHECK:
Substituting into the constraints, we have:

\[
\begin{align*}
3(10) - 6 &= 24 \leq 24 & \checkmark \\
10 + 6 &= 16 \leq 16 & \checkmark \\
5(10) + 4(6) &= 74 \geq 40 & \checkmark \\
-(10) + 2(6) &= 2 \leq 20 & \checkmark \\
\text{and} & \quad 10, 6 \geq 0 & \checkmark \\
\end{align*}
\]

(ii) We now want to maximise $P = 3x + 4y$. We have

\[
\begin{align*}
P(8, 0) &= 3(8) + 4(0) = 24 \\
P(0, 10) &= 3(0) + 4(10) = 40 \\
P(10, 6) &= 3(10) + 4(6) = 54 \\
P(4, 12) &= 3(4) + 4(12) = 60 \\
\end{align*}
\]

So the optimal solution is $(x, y) = (4, 12)$ giving the optimal value $P = 60$.

CHECK:
Substituting into the constraints, we have:

\[
\begin{align*}
3(4) - 12 &= 0 \leq 24 & \checkmark \\
4 + 12 &= 16 \leq 16 & \checkmark \\
5(4) + 4(12) &= 68 \geq 40 & \checkmark \\
-(4) + 2(12) &= 20 \leq 20 & \checkmark \\
\text{and} & \quad 4, 12 \geq 0 & \checkmark \\
\end{align*}
\]
(b) Let

\[ c \] be the number of batches of capsules manufactured (3000 per batch),
and \[ t \] be the number of batches of tablets manufactured (3000 per batch).

So \( c \) and \( t \) must be nonnegative integers.

From the worded problem we also observe the following:

- time constraint: \[ 3\frac{1}{2}c + 2\frac{1}{2}t \leq 25 \]
- resource constraints: \[ c \leq 5 \text{ and } t \leq 7 \]
- objective function (profit ($\$$)): \[ P = 175c + 150t \]

In summary, the linear programming problem is:

Maximise \[ P = 175c + 150t \]
subject to \[ 3\frac{1}{2}c + 2\frac{1}{2}t \leq 25 \]
\[ c \leq 5 \]
\[ t \leq 7 \]
\[ c, t \geq 0 \]