Out of 8 - Parts (a) and (b) each out of 4.

(a) (i) We have \( f(x) = x^3 - x^2 - x + 2 \). Stationary points of \( f(x) \) occur where \( f'(x) = 0 \). So
\[
 f'(x) = 3x^2 - 2x - 1 = (3x + 1)(x - 1) .
\]
So \( f'(x) = 0 \) when \( x = -\frac{1}{3} \) or \( x = 1 \). Thus the only stationary point within the interval \((-1, 0)\) is
\[
 x = -\frac{1}{3}.
\]
One way to determine the nature of this stationary point is to use the 2nd derivative test. We have \( f''(x) = 6x - 2 \), so
\[
 f''\left(-\frac{1}{3}\right) = 6\left(-\frac{1}{3}\right) - 2 = -4 < 0
\]
so by the 2nd derivative test, \( x = -\frac{1}{3} \) is a local maximum.

(ii) The absolute maximum and minimum values of \( f(x) \) over \([-1, 0]\) may occur at either a stationary point of \( f \) within the interval, an endpoint of the interval, or a point in the interval where \( f'(x) \) is undefined. Since \( f'(x) \) is defined for all \( x \), we compute
\[
 f\left(-\frac{1}{3}\right) = \left(-\frac{1}{3}\right)^3 - \left(-\frac{1}{3}\right)^2 - \left(-\frac{1}{3}\right) + 2 = \frac{59}{27} = 2\frac{5}{27}
\]
\[
 f(-1) = (-1)^3 - (-1)^2 - (-1) + 2 = 1
\]
and \( f(0) = 2 \).

Therefore

the absolute maximum value of \( f \) over \([-1, 0]\) is \( f\left(-\frac{1}{3}\right) = \frac{59}{27} \),

and the absolute minimum value of \( f \) over \([-1, 0]\) is \( f(-1) = 1 \).

(b) Let \( x \) (in centimeters) denote the side length of the square base of the box, and \( y \) (in centimeters) the height of the box. For the box to physically exist, \( x, y > 0 \). The volume of the box is
\[
 V = x^2 y = 500
\]
which implies \( y = \frac{500}{x^2} \). The surface area of the box is
\[
 S = x^2 + 4xy = x^2 + \frac{2000}{x}.
\]
We want to minimise \( S \) on the interval \( 0 < x < \infty \).
To find stationary points of \( S \) we compute:
\[
 \frac{dS}{dx} = 2x - \frac{2000}{x^2} .
\]
So the stationary points satisfy:

\[ 2x - \frac{2000}{x^2} = 0 \]

\[ \Rightarrow 2x = \frac{2000}{x^2} \]

\[ \Rightarrow x^3 = 1000 \]

\[ \Rightarrow x = 10. \]

We can use the second derivative test to check whether this is a minimum. We have

\[ \frac{d^2S}{dx^2} = 2 + \frac{4000}{x^3} \]

which is positive for all \( x > 0 \). Thus \( x = 10 \) is a local minimum point. Since \( \frac{dS}{dx} \) is defined for all \( x \in (0, \infty) \), \( x = 10 \) is the only critical point of \( S \) in this interval, and thus is also the global minimum.

When \( x = 10 \), \( y = 500/(10)^2 = 5 \), and so the dimensions of the box yielding minimum surface area are 10cm by 10cm by 5cm (the height being 5cm).

The minimum surface area is:

\[ S(10) = 10^2 + \frac{2000}{10} = 300 \text{ cm}^2. \]