

Adjoint Sensitivity Analysis for Attribution of Responsibility for Climate Change

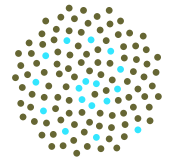
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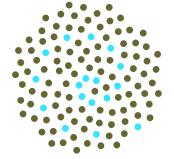


Acknowledgments



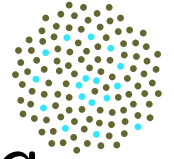
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- Collaborators: Cathy Trudinger and YingPing Wang of CSIRO Marine and Atmospheric Research and members of the MATCH working group on the Brazilian Proposal.

Summary

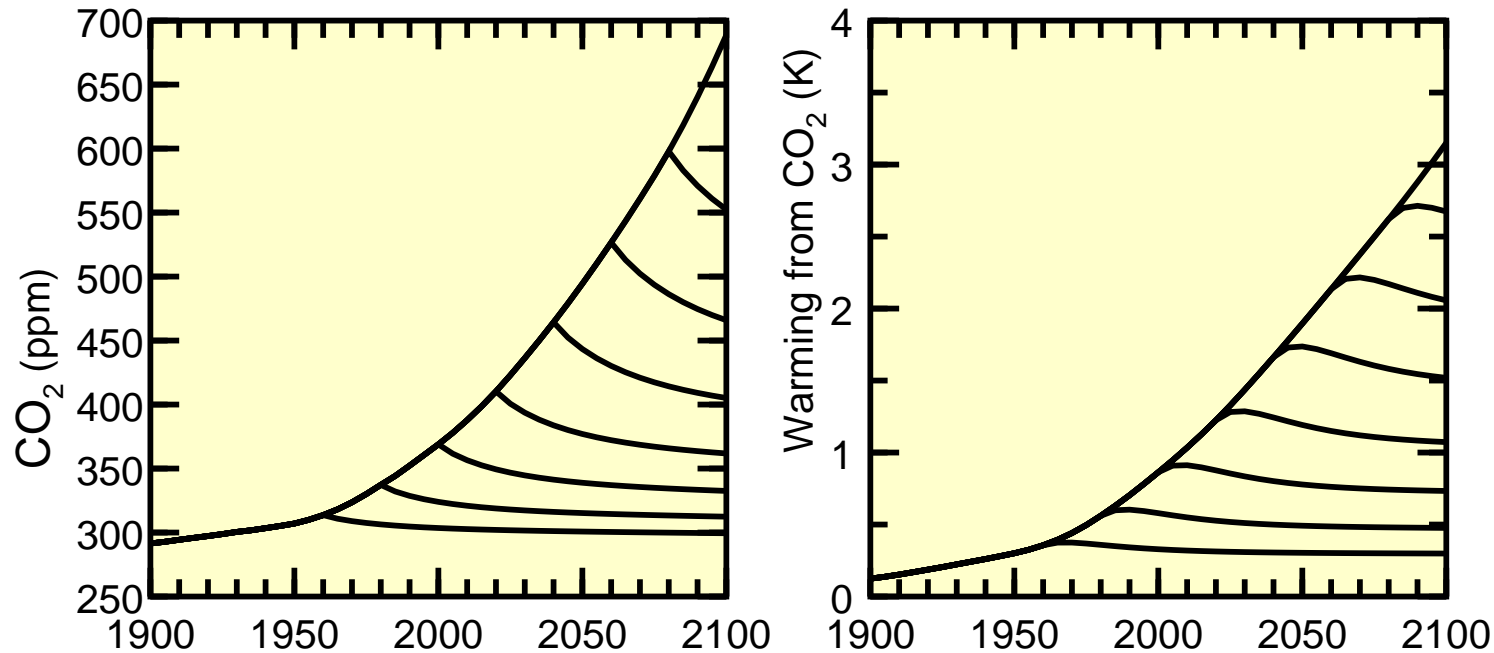


- Time-scales for the greenhouse effect
 - committed warming
- The Brazilian Proposal
 - setting reduction targets in proportion to responsibility
- Adjoint modelling
 - efficient calculation of sensitivities
- Analysing the Brazilian proposal
 - Who's to blame for the greenhouse effect?

Timescales



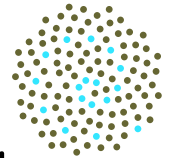
CO₂ concentrations and consequent warming, partitioned according to time of emission.



Lowest bands are from pre-1960 emissions, next from 1960 to 1980 emissions, etc.

Increase in contribution to warming after time of emissions from 'committed warming' effect.

Brazilian Proposal



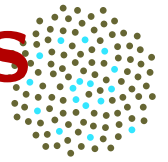
Tabled by Brazil during negotiations leading to Kyoto Protocol — Flicked-passed to Subsidiary Body for Scientific and Technical Advice (SBSTA).

Proposes that emission reduction targets should be proportional to nations' relative responsibility for the greenhouse effect.

Issues:

- Indicator? What quantity is used as a measure of the greenhouse effect?
- For what period of emissions is responsibility attributed?
- How are non-linear responses attributed?

Brazilian Proposal as Derivatives



As example, use indicator $T^* = T_{\text{CO}_2}(2000) =$ warming in 2000 from CO_2 emissions.

T^* is to be attributed to emissions $E_j(t)$ from country j with $E(t) = \sum_j E_j(t)$.

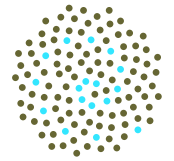
Differential attribution to country j of emissions at time t is

$$\frac{\partial T^*}{\partial E_j(t)} E_j(t) = \frac{\partial T^*}{\partial E(t)} E_j(t) = S(t) E_j(t)$$

where $S(t)$ is a Fréchet derivative.

Cumulated attribution: $T_j^* = \int S(t) E_j(t) dt$

Aims of adjoint modelling



Aim is to simplify calculations by separating parametric differentiation from model integration, expressed here in terms of Green's function \mathcal{G} of $\mathcal{L}\underline{u}(\cdot) = \underline{f}(\cdot)[\underline{a}]$.

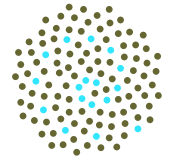
Considers $\nabla_{\underline{a}}\langle \underline{w}(\cdot) | \underline{u}(\cdot)[\underline{a}] \rangle$
where $\underline{u}(\cdot)[\underline{a}] = \mathcal{G}\underline{f}(\cdot)[\underline{a}]$ (or $\mathcal{L}\underline{u}(\cdot) = \underline{f}(\cdot)[\underline{a}]$)

Transforms as

$$\nabla_{\underline{a}}\langle \underline{w}(\cdot) | \underline{u}(\cdot)[\underline{a}] \rangle = \nabla_{\underline{a}}\langle \underline{w}(\cdot) | \mathcal{G}\underline{f}(\cdot)[\underline{a}] \rangle = \\ \nabla_{\underline{a}}\langle \mathcal{G}^\dagger \underline{w}(\cdot) | \underline{f}(\cdot)[\underline{a}] \rangle = \nabla_{\underline{a}}\langle \underline{v}(\cdot) | \underline{f}(\cdot)[\underline{a}] \rangle$$

where $\underline{v}(\cdot) = \mathcal{G}^\dagger \underline{w}(\cdot)$ defines a single function $\underline{v}(\cdot)$ with no dependence of \underline{a} .

Principles of adjoint modelling



Given $\underline{u}(\cdot)[\underline{a}] = \mathcal{G}\underline{f}(\cdot)[\underline{a}]$, where typically $\mathcal{L}\underline{u}(\cdot)$ is linearisation of a more general model:

Formally:

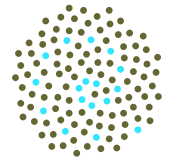
$$\begin{aligned}\nabla_{\underline{a}}\langle\underline{w}(\cdot)|\underline{u}(\cdot)[\underline{a}]\rangle &= \nabla_{\underline{a}}\langle\underline{w}(\cdot)|\mathcal{G}\underline{f}(\cdot)[\underline{a}]\rangle = \\ \nabla_{\underline{a}}\langle\mathcal{G}^\dagger\underline{w}(\cdot)|\underline{f}(\cdot)[\underline{a}]\rangle &= \nabla_{\underline{a}}\langle\underline{v}(\cdot)|\underline{f}(\cdot)[\underline{a}]\rangle \\ \text{with } \underline{v}(\cdot) &= \mathcal{G}^\dagger\underline{w}(\cdot)\end{aligned}$$

In practice, used as $\mathcal{L}\underline{u}(\cdot) = \underline{f}(\cdot)[\underline{a}]$

$$\begin{aligned}\nabla_{\underline{a}}\langle\underline{w}(\cdot)|\underline{u}(\cdot)[\underline{a}]\rangle &= \nabla_{\underline{a}}\langle\mathcal{L}^\dagger\underline{v}(\cdot)|\underline{u}(\cdot)[\underline{a}]\rangle = \\ \nabla_{\underline{a}}\langle\underline{v}(\cdot)|\mathcal{L}\underline{u}(\cdot)[\underline{a}]\rangle &= \nabla_{\underline{a}}\langle\underline{v}(\cdot)|\underline{f}(\cdot)[\underline{a}]\rangle\end{aligned}$$

with $\underline{w}(\cdot) = \mathcal{L}^\dagger\underline{v}(\cdot)$ giving equations for adjoint model.

Applying adjoint modelling



Differentiation (only case used in this talk)

$$\nabla_{\underline{a}} \langle \underline{w}(\cdot) | \mathcal{G} \underline{f}(\cdot) [\underline{a}] \rangle = \nabla_{\underline{a}} \langle \mathcal{G}^\dagger \underline{w}(\cdot) | \underline{f}(\cdot) [\underline{a}] \rangle$$

Gradients for soft constraints .

$$\begin{aligned} \nabla_{\underline{a}} \langle \underline{Hu} - \underline{z} | \underline{Hu} - \underline{z} \rangle &= 2 \nabla_{\underline{a}} \langle \underline{Hu}_0 - \underline{z} | \underline{Hu} \rangle = \\ 2 \nabla_{\underline{a}} \langle \underline{Hu}_0 - \underline{z} | \underline{H} \mathcal{L} \underline{f} \rangle &= 2 \nabla_{\underline{a}} \langle (\underline{H} \mathcal{L})^\dagger (\underline{Hu}_0 - \underline{z}) | \underline{f} \rangle \end{aligned}$$

Gradients, with hard constraints: $\mathcal{L} \underline{u}(\cdot) = 0$

$$\Theta^* = \Theta - \langle \underline{v}(\cdot) | \mathcal{L} \underline{u}(\cdot) \rangle$$

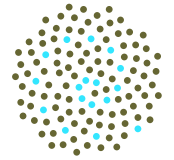
The function $\underline{v}(\cdot)$ is the Lagrange multiplier.

$$\nabla_{\underline{u}} \Theta^* = \nabla_{\underline{u}} \Theta - \nabla_{\underline{u}} \langle \mathcal{L}^\dagger | \underline{u}(\cdot) \rangle, \text{ whence}$$

$$\mathcal{L}^\dagger \underline{v}(\cdot) = \nabla_{\underline{u}} \Theta \text{ — the adjoint equations}$$

define the Lagrange multiplier

Tangent Linear Model (TLM)



For N DEs: $\frac{d}{dt}x_j = g_j(\{x_k\}, a, t)$ for $j = 1, N$

we can define sensitivities as

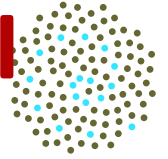
$$y_j = \frac{\partial}{\partial a} x_j \quad \text{for } j = 1, N \quad \text{or} \quad y_{j,p} = \frac{\partial}{\partial a_p} x_j$$

to give 'tangent linear model(s)':

$$\frac{d}{dt}y_m = \frac{\partial}{\partial a} g_m(\{x_k\}, a, t) + \sum_n \frac{\partial}{\partial x_n} g_m(\{x_k\}, a, t) y_n$$

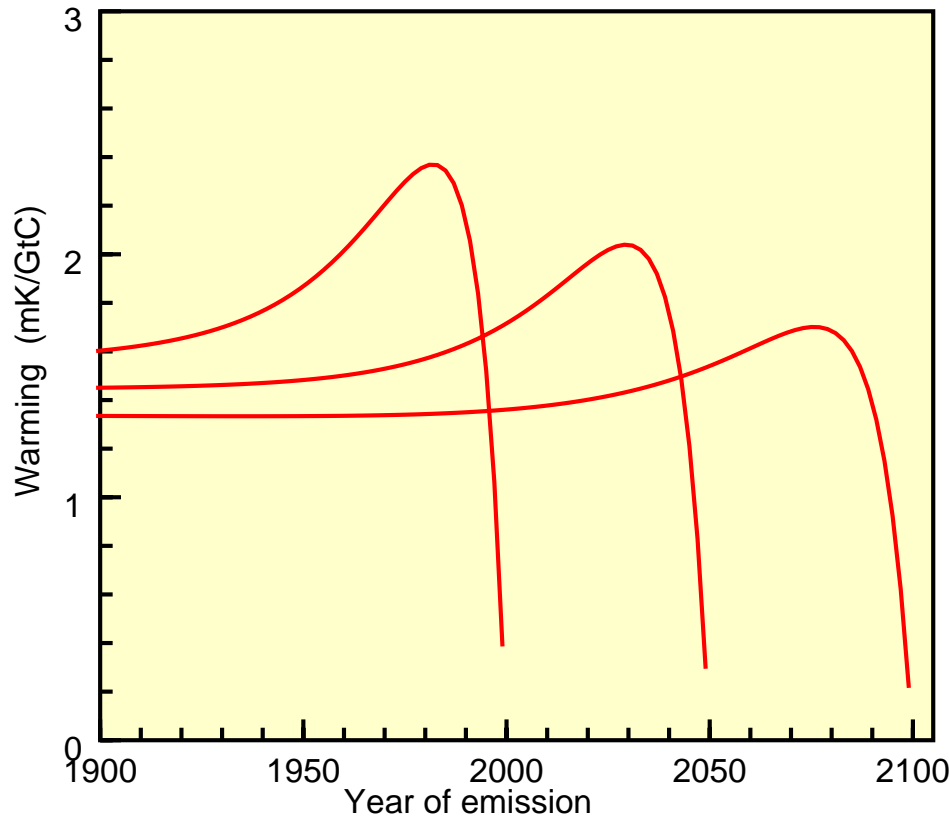
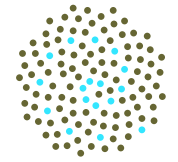
$$\frac{d}{dt}y_{m,p} = \frac{\partial}{\partial a_p} g_m(\{x_k\}, \underline{a}, t) + \sum_n \frac{\partial}{\partial x_n} g_m(\{x_k\}, \underline{a}, t) y_{n,p}$$

Analysing the Brazilian Proposal



- Construct simple climate model
- Construct linearisation (e.g. by automatic differentiation)
- Calculate sensitivities, either by brute force application of linearised model or by explicit adjoint model.
- Apply sensitivities to histories of emissions from each nation
- Repeat for all greenhouse gases

Results: Fréchet Derivatives

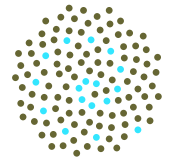


Assumes IS92a emissions. Represents temperature by response function. Linear responses for ocean and biotic carbon, coupled non-linearly to atmospheric CO₂ (as in CSIRO study).

$$\frac{\partial}{\partial E(t)} T(\tau) \text{ for } \tau = 2000, 2050, 2100.$$

Decrease as $t \rightarrow \tau$ shows 'committed warming'. At any time, warming from the most recent releases is yet to happen.

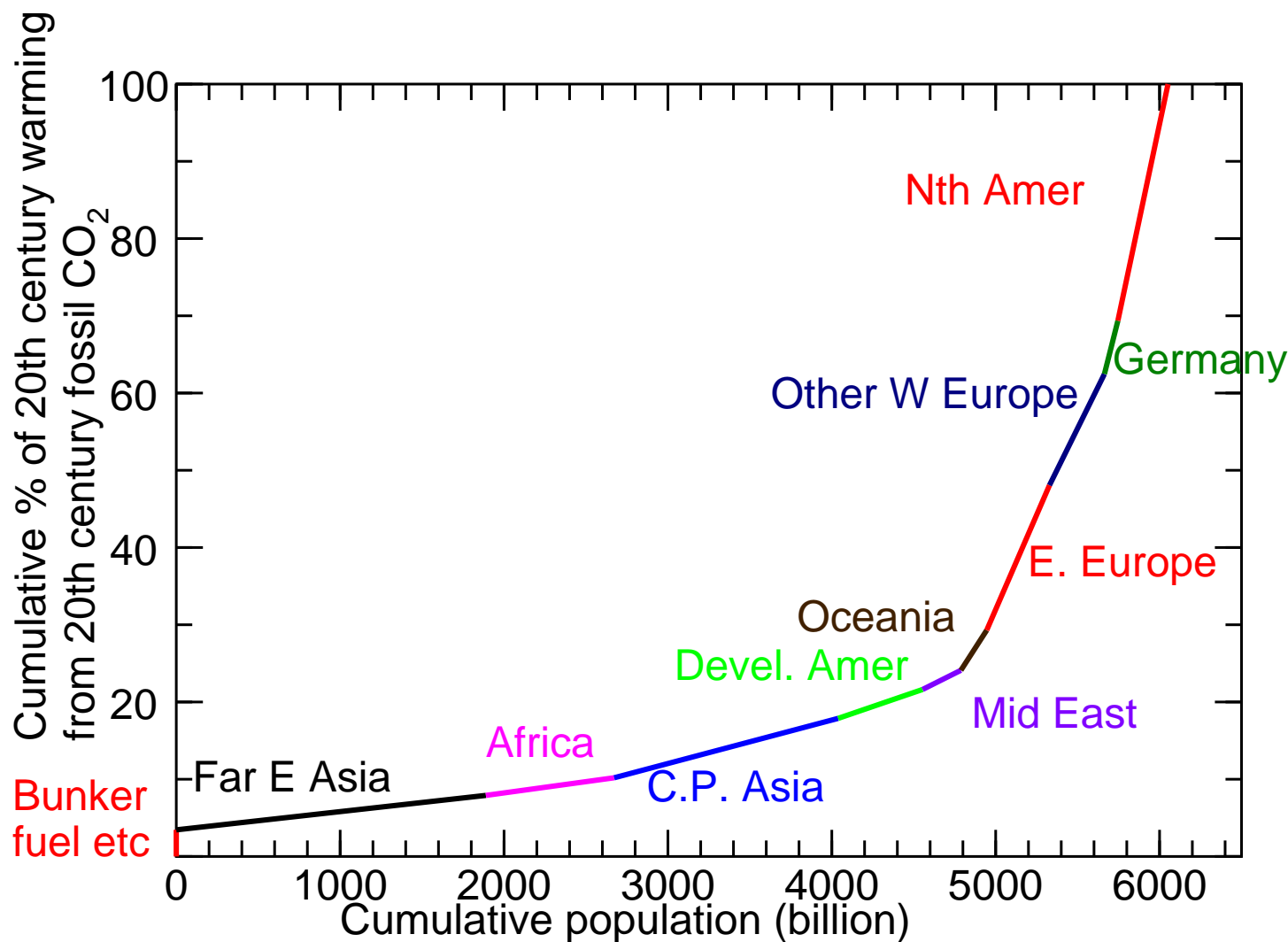
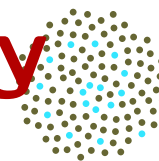
Implications



- For a given indicator, T^* , calculation of $S(t)$ allows attribution to any nation.
- $S(t)$ most efficiently calculated from adjoint model, but for multiple indicator times, tangent linear model not too inefficient.
- Sensitivity of T_j^* to model uncertainties can be obtained as second derivatives.
- Sensitivity of T_j^* to uncertainties in emissions can be obtained as

$$\text{Var}[T_j^*] = \int \int S(t) \text{Cov}[E_j(t), E_j(t')] S(t') dt' dt$$

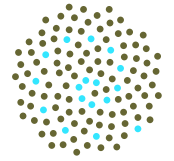
Attribution 2000, fossil CO₂ only



Cumulative responsibility for the fossil CO₂ component of warming vs cumulative population.

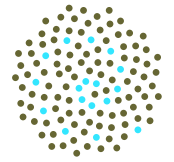
In preparation for ANZIAM 2007

Concluding remarks



- An interesting example of adjoint sensitivity analysis and automatic differentiation
- The Brazilian Proposal is on the agenda, for formal consideration by Conference of Parties (to the Climate Change Convention) in 2008
- Expert working group is extending calculations to include all major greenhouse gases, with detailed national attribution

Further Information



Andreas Griewank, 2000, *Evaluating Derivatives: Principles and Techniques of Algorithmic Differentiation*, (SIAM, Philadelphia).

MATCH website (Brazilian Proposal):
<http://www/match-info.net>

I.G. Enting, 2005, *Automatic differentiation in the analysis of strategies for mitigation of global change*, International Congress on Modelling and Simulation, Melbourne, 2005. Ed. A. Zerger and R. M. Argent, 7pp
<http://www.mssanz.org.au/modsim05/papers/enting.pdf>