Adjoint Sensitivity Analysis for Attribution of Responsibility for Climate Change

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Summary

• Time-scales for the greenhouse effect
  – committed warming

• The Brazilian Proposal
  – setting reduction targets in proportion to responsibility

• Adjoint modelling
  – efficient calculation of sensitivities

• Analysing the Brazilian proposal
  – Who’s to blame for the greenhouse effect?

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Timescales

$\text{CO}_2$ concentrations and consequent warming, partitioned according to time of emission.

Lowest bands are from pre-1960 emissions, next from 1960 to 1980 emissions, etc. Increase in contribution to warming after time of emissions from ‘committed warming’ effect.

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Brazilian Proposal

Tabled by Brazil during negotiations leading to Kyoto Protocol — Flicked-passed to Subsidiary Body for Scientific and Technical Advice (SBSTA).
Proposes that emission reduction targets should be proportional to nations’ relative responsibility for the greenhouse effect.

Issues:

- Indicator? What quantity is used as a measure of the greenhouse effect?
- For what period of emissions is responsibility attributed?
- How are non-linear responses attributed?

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Brazilian Proposal as Derivatives

As example, use indicator $T^* = T_{CO2}(2000) =$ warming in 2000 from CO$_2$ emissions. $T^*$ is to be attributed to emissions $E_j(t)$ from country $j$ with $E(t) = \sum_j E_j(t)$.

Differential attribution to country $j$ of emissions at time $t$ is

$$\frac{\partial T^*}{\partial E_j(t)} E_j(t) = \frac{\partial T^*}{\partial E(t)} E_j(t) = S(t)E_j(t)$$

where $S(t)$ is a Fréchet derivative.

Cumulated attribution: $T^*_j = \int S(t)E_j(t) \, dt$
Aims of adjoint modelling

Aim is to simplify calculations by separating parametric differentiation from model integration, expressed here in terms of Green’s function $G$ of $Lu(\cdot) = f(\cdot)[a]$.

Considers $\nabla_a \langle w(\cdot) | u(\cdot)[a] \rangle$
where $u(\cdot)[a] = Gf(\cdot)[a]$ (or $Lu(\cdot) = f(\cdot)[a]$)

Transforms as
$\nabla_a \langle w(\cdot) | u(\cdot)[a] \rangle = \nabla_a \langle w(\cdot) | Gf(\cdot)[a] \rangle = \nabla_a \langle G^\dagger w(\cdot) | f(\cdot)[a] \rangle = \nabla_a \langle v(\cdot) | f(\cdot)[a] \rangle$
where $v(\cdot) = G^\dagger w(\cdot)$ defines a single function $v(\cdot)$ with no dependence of $a$.

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Principles of adjoint modelling

Given $u(.)[a] = Gf(.)[a]$, where typically $Lu(.)$ is linearisation of a more general model:

Formally:

$$\nabla_a \langle w(.)|u(.)[a] \rangle = \nabla_a \langle w(.)|Gf(.)[a] \rangle = \nabla_a \langle G^\dagger w(.)|f(.)[a] \rangle = \nabla_a \langle v(.)|f(.)[a] \rangle$$

with $v(.) = G^\dagger w(.)$

In practice, used as $Lu(.) = f(.)[a])$

$$\nabla_a \langle w(.)|u(.)[a] \rangle = \nabla_a \langle L^\dagger v(.)|u(.)[a] \rangle = \nabla_a \langle v(.)|Lu(.)[a] \rangle = \nabla_a \langle v(.)|f(.)[a] \rangle$$

with $w(.) = L^\dagger v(.)$ giving equations for adjoint model.

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Applying adjoint modelling

Differentiation (only case used in this talk)
\[ \nabla_a \langle \omega(.) | Gf(.)[a] \rangle = \nabla_a \langle G^\dagger \omega(.) | f(.)[a] \rangle \]

Gradients for soft constraints.
\[ \nabla_a \langle Hu - z | Hu - z \rangle = 2 \nabla_a \langle Hu_0 - z | Hu \rangle = 2 \nabla_a \langle Hu_0 - z | H\mathcal{L}f \rangle = 2 \nabla_a \langle (H\mathcal{L})^\dagger (Hu_0 - z) | f \rangle \]

Gradients, with hard constraints: \( \mathcal{L}u(.) = 0 \)
\[ \Theta^* = \Theta - \langle v(.) | \mathcal{L}u(.) \rangle \]

The function \( v(.) \) is the Lagrange multiplier.
\[ \nabla_u \Theta^* = \nabla_u \Theta - \nabla_u \langle \mathcal{L}^\dagger | u(.) \rangle, \text{ whence} \]
\[ \mathcal{L}^\dagger v(.) = \nabla_u \Theta \text{ — the adjoint equations} \]

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Tangent Linear Model (TLM)

For $N$ DEs: 

$$ \frac{d}{dt} x_j = g_j(\{x_k\}, a, t) \quad \text{for } j = 1, N $$

we can define sensitivities as

$$ y_j = \frac{\partial}{\partial a} x_j \quad \text{for } j = 1, N \quad \text{or} \quad y_{j,p} = \frac{\partial}{\partial a_p} x_j $$

to give ‘tangent linear model(s)’:

$$ \frac{d}{dt} y_m = \frac{\partial}{\partial a} g_m(\{x_k\}, a, t) + \sum_n \frac{\partial}{\partial x_n} g_m(\{x_k\}, a, t) \ y_n $$

$$ \frac{d}{dt} y_{m,p} = \frac{\partial}{\partial a_p} g_m(\{x_k\}, a, t) + \sum_n \frac{\partial}{\partial x_n} g_m(\{x_k\}, a, t) \ y_{n,p} $$

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Analysing the Brazilian Proposal

- Construct simple climate model
- Construct linearisation (e.g. by automatic differentiation)
- Calculate sensitivities, either by brute force application of linearised model or by explicit adjoint model.
- Apply sensitivities to histories of emissions from each nation
- Repeat for all greenhouse gases

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Results: Fréchet Derivatives

Assumes IS92a emissions.
Represents temperature by response function. Linear responses for ocean and biotic carbon, coupled non-linearly to atmospheric CO$_2$ (as in CSIRO study).

\[
\frac{\partial}{\partial E(t)} T(\tau) \text{ for } \tau = 2000, 2050, 2100.
\]

Decrease as $t \rightarrow \tau$ shows ‘committed warming’.
At any time, warming from the most recent releases is yet to happen.

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Implications

• For a given indicator, $T^*$, calculation of $S(t)$ allows attribution to any nation.

• $S(t)$ most efficiently calculated from adjoint model, but for multiple indicator times, tangent linear model not too inefficient.

• Sensitivity of $T^*_j$ to model uncertainties can be obtained as second derivatives.

• Sensitivity of $T^*_j$ to uncertainties in emissions can be obtained as

$$\text{Var}[T^*_j] = \int \int S(t) \text{Cov}[E_j(t), E_j(t')] S(t') dt' dt$$
Cumulative responsibility for the fossil CO$_2$ component of warming vs cumulative population.

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Concluding remarks

• An interesting example of adjoint sensitivity analysis and automatic differentiation

• The Brazilian Proposal is on the agenda, for formal consideration by Conference of Parties (to the Climate Change Convention) in 2008

• Expert working group is extending calculations to include all major greenhouse gases, with detailed national attribution
Further Information


MATCH website (Brazilian Proposal):
http://www/match-info.net


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