Attribution of Responsibility for Climate Change

The Mathematics Behind the Brazilian Proposal

Ian G. Enting

MASCOS

The University of Melbourne
Acknowledgments

- The Center of Excellence for Mathematics and Statistics of Complex Systems (MASCOS) is funded by the Australian Research Council (ARC).
- My fellowship at MASCOS is supported by CSIRO through a sponsorship agreement.
- Collaborators: Cathy Trudinger of CSIRO Marine and Atmospheric Research and members of the MATCH working group on the Brazilian Proposal.
Summary

• Time-scales for the greenhouse effect
  – committed warming

• The Brazilian Proposal
  – setting reduction targets in proportion to responsibility

• Adjoint modelling & automatic differentiation
  – efficient calculation of sensitivities
  – operator overloading

• Analysing the Brazilian proposal
  – Who’s to blame for the greenhouse effect?
The causal chain

From IPCC Third Assessment report
IPCC is Cautious

- The IPCC requirement for a consensus evaluation of ‘well-established science’ can mean that IPCC reports lag behind the forefront of science, sometimes by more than a decade.
- Similarly, Al Gore’s book and film are very careful to avoid premature speculation about extreme possibilities.
- Recent study (Science Express, 1/2/07) indicates that changes since 2001 are tracking the high end of IPCC projections.
- Overview by Barrie Pittock on mechanisms that may imply current under-estimation of change.
- For ‘alarmist’ views, see ‘Pentagon scenarios’.
Timescales

\( \text{CO}_2 \) concentrations and consequent warming, partitioned according to time of emission.

Lowest bands are from pre-1960 emissions, next from 1960 to 1980 emissions, etc. Increase in contribution to warming after time of emissions from ‘committed warming’ effect.
Some key dates

- circa 1960: Keeling measures CO₂ growth
- Oct 1985. *it is now believed that in the first half of the next century a rise of global mean temperature could occur which is greater than any in man’s history.* (UNEP/WMO/ICSU conference, Villach).
- 1998. IPCC established by WMO and UNEP
- 1995. CoP-1, Berlin Mandate (to negotiate a protocol)
- Kyoto Protocol: 11/12/97. came into force 16/2/05.
- 2001 (IPCC) balance of evidence suggests discernable human influence
- Feb 2007 (IPCC): *unequivocal* attribution to human agency of most of the warming since 1950.
Mitigation frameworks

- Kyoto, specified percentage reductions
- Contraction and convergence
- Sector-based targets – e.g. the Tryptique framework underlying target-setting within the ‘EU bubble’

Note that measures such as carbon trading, clean development mechanism or the McKibbin-Wilcoxen multi-level permits are not primarily about mitigation levels, they are about economically-efficient implementation.
Brazilian Proposal

Tabled by Brazil during negotiations leading to Kyoto Protocol — Flicked-passed to Subsidiary Body for Scientific and Technical Advice (SBSTA).

Proposes that emission reduction targets should be proportional to nations’ relative responsibility for the greenhouse effect.

Issues:

• Indicator? What quantity is used as a measure of the greenhouse effect?

• For what period of emissions is responsibility attributed?

• How are non-linear responses attributed?
Political significance

- It captures the historical responsibility of developed nations (for global warming) and the requirement of developed nations to take the lead in combating climate change — i.e. as prescribed by the UN Framework Convention on Climate Change.

- It provides a formula that can apply to all nations

- Therefore it provides a way of engaging developing nations as they develop.
Expert Working Group: MATCH

- Initially under the auspices of SBSTA, initial meeting in Brazil
- Joint paper published:
Brazilian Proposal as Derivatives

As example, use indicator $T^* = T_{\text{CO}_2}(2000)$ = warming in 2000 from CO$_2$ emissions. $T^*$ is to be attributed to emissions $E_j(t)$ from country $j$ with $E(t) = \sum_j E_j(t)$.

Differential attribution to country $j$ of emissions at time $t$ is

$$\frac{\partial T^*}{\partial E_j(t)} E_j(t) = \frac{\partial T^*}{\partial E(t)} E_j(t) = S(t)E_j(t)$$

where $S(t)$ is a Fréchet derivative.

Cumulated attribution: $T^*_j = \int S(t)E_j(t) \, dt$
Aims of adjoint modelling

Aim is to simplify calculations by separating parametric differentiation from model integration, expressed here in terms of Green’s function $G$ of $ Lu(.) = f(.)[a] $. Considers $ \nabla_a \langle w(.) | u(.)[a] \rangle $ where $ u(.)[a] = G f(.)[a] $ (or $ Lu(.) = f(.)[a] $)

Transforms as

$ \nabla_a \langle w(.) | u(.)[a] \rangle = \nabla_a \langle w(.) | G f(.)[a] \rangle = \nabla_a \langle G^\dagger w(.) | f(.)[a] \rangle = \nabla_a \langle v(.) | f(.)[a] \rangle $ where $ v(.) = G^\dagger w(.) $ defines a single function $ v(.) $ with no dependence on $ a $. 
Principles of adjoint modelling

Given \( u(.)[a] = Gf(.)[a] \), where typically \( \mathcal{L}u(.) \) is linearisation of a more general model:

Formally:
\[
\nabla_a \langle w(.)|u(.)[a] \rangle = \nabla_a \langle w(.)|Gf(.)[a] \rangle = \nabla_a \langle G^\dagger w(.)|f(.)[a] \rangle = \nabla_a \langle v(.)|f(.)[a] \rangle
\]
with \( v(.) = G^\dagger w(.) \)

In practice, used as \( \mathcal{L}u(.) = f(.)[a] \)
\[
\nabla_a \langle w(.)|u(.)[a] \rangle = \nabla_a \langle \mathcal{L}^\dagger v(.)|u(.)[a] \rangle = \nabla_a \langle v(.)|\mathcal{L}u(.)[a] \rangle = \nabla_a \langle v(.)|f(.)[a] \rangle
\]
with \( w(.) = \mathcal{L}^\dagger v(.) \) giving equations for adjoint model.
Applying adjoint modelling

**Differentiation** (only case used in this talk)
\[ \nabla_a \langle w(.)|Gf(.)[a]\rangle = \nabla_a \langle G^\dagger w(.)|f(.)[a]\rangle \]

**Gradients for soft constraints**
\[ \nabla_a \langle Hu - z|Hu - z\rangle = 2\nabla_a \langle Hu_0 - z|Hu\rangle = \\
2\nabla_a \langle Hu_0 - z|HLf\rangle = 2\nabla_a \langle (HL)^\dagger(Hu_0 - z)|f\rangle \]

**Gradients, with hard constraints:** \( \mathcal{L}_u(.) = 0 \)
\[ \Theta^* = \Theta - \langle v(.)|\mathcal{L}u(.)\rangle \]

The function \( v(.) \) is the Lagrange multiplier.
\[ \nabla_u \Theta^* = \nabla_u \Theta - \nabla_u \langle \mathcal{L}^\dagger|u(.)\rangle, \text{ whence} \]
\[ \mathcal{L}^\dagger v(.) = \nabla_u \Theta \] — the adjoint equations define the Lagrange multiplier
Tangent Linear Model (TLM)

For $N$ DEs: \[ \frac{d}{dt} x_j = g_j(\{x_k\}, a, t) \quad \text{for } j = 1, N \]

we can define sensitivities as

\[ y_j = \frac{\partial}{\partial a} x_j \quad \text{for } j = 1, N \]

or \[ y_{j,p} = \frac{\partial}{\partial a_p} x_j \]

to give ‘tangent linear model(s)’:

\[ \frac{d}{dt} y_m = \frac{\partial}{\partial a} g_m(\{x_k\}, a, t) + \sum_n \frac{\partial}{\partial x_n} g_m(\{x_k\}, a, t) \; y_n \]

\[ \frac{d}{dt} y_{m,p} = \frac{\partial}{\partial a_p} g_m(\{x_k\}, a, t) + \sum_n \frac{\partial}{\partial x_n} g_m(\{x_k\}, a, t) \; y_{n,p} \]
Algorithmic Differentiation (AD)

Differentiation by successive use of chain rule. For binary operation $c = f(a, b)$,

$$\frac{\partial c}{\partial \alpha} = \frac{\partial f}{\partial a} \cdot \frac{\partial a}{\partial \alpha} + \frac{\partial f}{\partial b} \cdot \frac{\partial b}{\partial \alpha}$$

e.g. $c = a + b \rightarrow \frac{\partial c}{\partial \alpha} = \frac{\partial a}{\partial \alpha} + \frac{\partial b}{\partial \alpha}$

or $c = a \times b \rightarrow \frac{\partial c}{\partial \alpha} = b \cdot \frac{\partial a}{\partial \alpha} + a \cdot \frac{\partial b}{\partial \alpha}$

Automatically derives TLM: converting program to code for derivatives, one operation at a time.
Approaches to AD

Hand-code program to calculate derivatives — laborious, error-prone and must be repeated each time the model changes.

Symbolic algebra (e.g. Mathematica) — problematic for adjoints.

Tangent/adjoint compilers — transform source into code for tangent or adjoint models.

Operator overloading to produce a ‘script’ that is analysed to give code for the derivatives.

Use operator overloading capabilities directly — straightforward for tangent-linear-model, but restricted applicability to adjoint models.
AD by operator overloading

- Needs object-oriented language, C++ or F90
- Define new types that include derivative information
- Use overloading to define operations on these types
- Modify type declarations in original model code to invoke new types
- Set up requisite I/O for derivative information
Analysing the Brazilian Proposal

- Construct simple climate model
- Construct linearisation (e.g. by automatic differentiation)
- Calculate sensitivities, either by brute force application of linearised model or by explicit adjoint model.
- Apply sensitivities to histories of emissions from each nation
- Repeat for all greenhouse gases
Results: Fréchet Derivatives

Assumes IS92a emissions. Represents temperature by response function. Linear responses for ocean and biotic carbon, coupled non-linearly to atmospheric CO$_2$ (as in CSIRO study).

\[ \frac{\partial}{\partial E(t)} T(\tau) \] for \( \tau = 2000, 2050, 2100 \).

Decrease as \( t \to \tau \) shows ‘committed warming’. At any time, warming from the most recent releases is yet to happen.
Implications

• For a given indicator, $T^*$, calculation of $S(t)$ allows attribution to any nation.

• $S(t)$ most efficiently calculated from adjoint model, but for multiple indicator times, tangent linear model not too inefficient.

• Sensitivity of $T^*_j$ to model uncertainties can be obtained as second derivatives.

• Sensitivity of $T^*_j$ to uncertainties in emissions can be obtained as

\[
\text{Var}[T^*_j] = \int \int S(t) \text{Cov}[E_j(t), E_j(t')] S(t') \, dt' \, dt
\]
Cumulative responsibility for the fossil CO$_2$ component of warming vs cumulative population.
Concluding remarks

• An interesting example of adjoint sensitivity analysis and automatic differentiation

• The Brazilian Proposal is on the agenda, for formal consideration by Conference of Parties (to the Climate Change Convention) in 2008

• Expert working group is extending calculations to include all major greenhouse gases, with detailed national attribution
Further Information

MATCH website (Brazilian Proposal):
http://www/match-info.net


My AD talk from Berkeley is on my website.
Fragment of C++ class definition to implement operator overloading:

class Xvar{
public:
static const int ns = _NUMDERIVS+1;
double xs[_NUMDERIVS+1];
Xvar operator*(Xvar b);
...}

Xvar Xvar::operator*(Xvar b) {
Xvar c;
for (int i=1; i < ns; i++)
    c.xs[i] = xs[i]*b.xs[0]+xs[0]*b.xs[i];
c.xs[0] = xs[0]*b.xs[0];
return c; }
...
Usage

Original

```c
double F_co2(double c) {
    double a;
    a = log(c/280.0)*5.35;
    return a;
}
```

```c
... double cc;
...
ff = F_CO2(cc)
```

Transformed

```c
Xvar F_co2(Xvar c) {
    Xvar a;
    a = log(c/280.0)*5.35;
    return a;
}
```

```c
... Xvar cc;
// Derivatives wrt
// initial value of cc
cc.set(280,1);
...
ff = F_CO2(cc)
```