

Department of Mathematics and Statistics
University of Melbourne
620-221: Real and Complex Analysis
Mid-Semester Test, 23 April 2008.

- (1) Find the modulus and argument of

$$\frac{(2i)^3}{(1+i)^5}.$$

- (2) Sketch, on the complex plane, the following set of complex numbers:

$$\{z \in \mathbb{C} : |z - i| < 1, |z - 1| < 1\}.$$

Write down a complex number which is in the set.

- (3) Is the set $\{z : |z| > 1, \operatorname{Re}(z) \leq 0\}$ an open subset of the complex plane? Give careful reasons for your answer.
- (4) Show, *without using the Heine-Borel theorem* (that is, directly from the definition of compact), that the real line is not a compact set.
- (5) For what real values of a and b is the function f given by

$$f(x + iy) = (ax^2 + y^2) + i(bxy)$$

an entire function of $z = x + iy$?

- (6) If the real part of an entire function of $z = x + iy$ is given by $x^2 - y^2 + 2y$, what can the imaginary part be?
- (7) For what z does the power series

$$\sum_{n=0}^{\infty} \frac{z^n}{2^{n^2}} = 1 + \frac{z}{2} + \frac{z^2}{16} + \frac{z^3}{2^9} + \dots$$

converge?

- (8) Expand the function $f(z) = \frac{z}{1+3z}$ as a power series centered at $z = 0$. What is the radius of convergence of this power series?
- (9) Suppose that f is represented by a power series, centered at 0 and with radius of convergence 2. Suppose also that $f(1/n) = 1$ for $n = 1, 2, 3, 4, \dots$. Write down $f(i)$ and give a brief explanation for your answer.
- (10) Find the real and imaginary parts of $\cos(i)$.