Department of Mathematics and Statistics
620-221: Real and Complex Analysis 2008
Assignment 1

Solutions to this assignment should be returned by 9.00am on Wednesday 2 April.

(1) Solve $z^2 - (6+5i)z + (2+16i) = 0$.

(2) Describe the projections onto the Riemann sphere of the following subsets of the complex plane. (Geographical descriptions, such as 'North Pole', 'line of latitude' etc, are acceptable as descriptions.)

  (a) the real and imaginary axes;
  (b) the unit disk—$\{z : |z| \leq 1\}$;
  (c) the right half plane—$\{z : \mathrm{Re}(z) > 0\}$;
  (d) the annulus—$\{z : 1 < |z| < 2\}$;
  (e) the exterior of the unit disc—$\{z : |z| > 1\}$.

(3) Show that the function defined on the complex numbers by

$$f(z) = \frac{z}{1 + |z|}$$

is continuous. (You should prove from first principles that the absolute value $z \mapsto |z|$ is a continuous function. You should then quote standard results on continuous functions rather than continue to use first principles.)

What is the image, under f, of the closed right half plane—that is, $\{z : \mathrm{Re}(z) \geq 0\}$. Explain why this shows that the image of a closed set under a continuous mapping need not be closed.