

Department of Mathematics and Statistics
620-221: Real and Complex Analysis, 2007

Exercises 5: Elementary Functions

(1) Write each of the following numbers in the form $x + iy$ with x and y real:

- (a) $\exp(2 + \frac{i\pi}{4})$ (d) $\sin 2i$ (g) $\cosh i$
(b) $\frac{1}{\exp(2 + i\pi)}$ (e) $\cos(1 - i)$ (h) $\sinh(1 + i\pi)$
(c) $\frac{\exp(1 + 3i\pi)}{\exp(-1 + \frac{i\pi}{2})}$ (f) $\tan(1 + i)$ (*longer*) (i) $\cosh(\frac{i\pi}{2})$

(2) Differentiate the following functions:

- (a) $\exp(\pi z^2)$ (c) $\frac{\sinh(z + 2)}{\exp(z^3)}$ (e) $\sin z \cosh z \exp z$
(b) $i \sin(1/z)$ (d) $\exp(\sin(2z))$ (f) $(\sinh z + 1)^2$

(3) Draw the following paths:

- (a) $z(t) = 1 + i + 2e^{it}$ ($0 \leq t \leq 2\pi$)
(b) $z(t) = t + i \cosh t$ ($-1 \leq t \leq 1$)
(b) $z(t) = \cosh t + i \sinh t$ ($-1 \leq t \leq 1$)

(4) Prove that the function $\exp(\bar{z})$ is not analytic anywhere.

(5) Find all of the values, and distinguish the principal value, of the following:

- (a) $\log i$ (c) $\log(\sqrt{3} + i)$ (e) $2^{\pi i}$
(b) $\log(1 - i)$ (d) i^{2i} (f) $(1 + i)^{1+i}$

(6) Solve the following equations:

- (a) $\exp(z) = 2i$ (b) $\text{Log}(z^2 - 1) = \frac{i\pi}{2}$ (c) $e^{2z} + e^z + 1 = 0$.