

The University of Melbourne
Semester Examination, June 2000
Department of Mathematics
Subject Number: 620-221
Subject Title : Real and Complex Analysis

Exam Duration: Three (3) hours
Reading Time: Fifteen (15) minutes
This paper has three (3) pages

Authorized materials:
None.

Instructions to Invigilators:
Calculators are not allowed. Examination paper may be retained.

Instructions to Students:
Full marks can be obtained by correctly answering all questions. Total marks 80.

Lodging of Paper with Baillieu Library:
This paper is to be lodged with the Baillieu library.

Beginning of exam paper

- 1 (a) Define the notion of a domain in the complex plane \mathbf{C} and decide whether the following subsets of \mathbf{C} are domains:

(1) $\{z : 3 \leq |z| < 4\}$, (2) the upper half plane $\{z : z = x + iy, y > 0\}$.

Indicate the reasons for your answers.

[6]

- (b) State Cauchy Convergence Criterion. Using it or otherwise, show that a series

$$\sum_{n=0}^{\infty} a_n$$

of complex numbers converges if

$$\sum_{n=0}^{\infty} |a_n|$$

converges.

[7]

- 2 (a) State the Cauchy-Riemann equations.

[3]

- (b) Show that if $f(z)$ is analytic in a domain and if $|f(z)|$ is constant, then $f(z)$ is constant in that domain.

[10]

- 3 (a) Determine the radius of convergence of the following power series:

$$(1) \sum_{n=0}^{\infty} \frac{z^n}{n!}, \quad (2) \sum_{n=0}^{\infty} (3+i)^n z^n.$$

[6]

- (b) Show that

$$\text{Log}(1+z) = z - \frac{z^2}{2} + \frac{z^3}{3} - \dots$$

for $|z| < 1$, where $\text{Log}(z)$ denotes the principal value of the logarithmic function. (Hint: Differentiate both sides.)

[7]

4 (a) State Cauchy's Integral Formula and evaluate

$$\int_{\gamma} \frac{\cos z}{z(z^2 + 8)} dz,$$

where n a positive integer and γ is the contour given by $\gamma(t) = 2e^{it}$, $0 \leq t \leq 2\pi$.

[7]

(b) State and prove Liouville's Theorem on bounded entire functions.

[6]

5 (a) Find the Taylor series of

$$\frac{1}{(z-1)(z-2)}$$

near 0.

[7]

(b) Evaluate

$$\int_{\gamma} \frac{e^{2z}}{(z-1)^{n+1}}$$

where γ is the contour given by $\gamma(t) = 2e^{it}$, $0 \leq t \leq 2\pi$.

[7]

6 (a) Find the Laurent expansion of

$$\frac{(z+1)}{z(z-4)^3}$$

at 4.

[7]

(b) Evaluate

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2+4)^3}$$

[7]

End of exam paper

