

The University of Melbourne
Semester Examination, Corrected version, June 2001
Department of Mathematics
Subject Number: 620-221
Subject Title : Real and Complex Analysis

Exam Duration: Three (3) hours
Reading Time: Fifteen (15) minutes
This paper has three (4) pages

Authorized materials:
None.

Intructions to Invigilators:
Calculators are not allowed. Examination paper may be retained.

Instuctions to Students:
Full marks can be obtained by correctly answering all questions. Total marks 80.

Lodging of Paper with Baillieu Library:
This paper is to be lodged with the Baillieu library.

Beginning of exam paper

1 (a) Define the notion of a domain in the complex plane \mathbf{C} and decide whether the following subsets of \mathbf{C} are domains:

(1) $\{z : |z| < 4\}$,

(2) the cut plane \mathbf{C}_π (recall that the cut plane consists of all points of \mathbf{C} other than the negative real numbers and zero).

Indicate the reasons for your answers.

[6]

(b) Give an example of a series

$$\sum_{n=0}^{\infty} a_n$$

of complex numbers which converges but

$$\sum_{n=0}^{\infty} |a_n|$$

does not converge. Indicate the reasons for your answer.

[4]

(c) Show that if $\sum_{n=0}^{\infty} a_n$ converges, then a_n converges to zero.

[3]

2 (a) Decide at which points (in their domain of definition) the following functions are complex differentiable.

(1)

$$f(z) = \frac{1}{\bar{z}}$$

where \bar{z} denotes the complex conjugate of z , and

(2) $f(z) = x^3 + 3xy^2 - 3x + i(y^3 + 3x^2y - 3y)$, where $z = x + iy$.

[8]

(b) Show that if $f(z)$ is analytic in a domain and if $f(z)$ is real, then $f(z)$ is constant in that domain.

[6]

3 (a) Determine the radius of convergence of the following power series:

$$(1) \sum_{n=0}^{\infty} \frac{z^n}{n^{\frac{1}{n}}}, \quad (2) \sum_{n=0}^{\infty} (3 - (-1)^n)^n z^n.$$

[6]

(b) The exponential function e^z is defined to be

$$e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} - \dots$$

for z . Show that $e^{z_1+z_2} = e^{z_1}e^{z_2}$ for all complex numbers z_1, z_2 (Quote the theorems that you use.)

[5]

4 (a) State Cauchy's Integral Formula and evaluate

$$\int_{\gamma} \frac{1}{z(1-z^2)} dz,$$

where γ is the contour given by $\gamma(t) = \frac{1}{2}e^{it}$, $0 \leq t \leq 2\pi$.

[5]

(b) Decide whether the function $f(z)$, where

$$f(z) = \frac{1}{z(1-z^2)}$$

has an antiderivative in the domain $\{z : 0 < |z| < 1\}$.

Explain reasons for your answer.

[5]

(c) State and prove Liouville's Theorem on bounded entire functions.

[5]

