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The University of Melbourne
Semester Examination, June 1998
Department of Mathematics
Subject Number: 620-221
Subject Title : Real and Complex Analysis

Exam Duration: Three (3) hours
Reading Time: Fifteen (15) minutes
This paper has three (3) pages

Authorized materials:
None.

Instructions to Invigilators:
Calculators are not allowed. Examination paper may be retained.

Instructions to Students:
Full marks can be obtained by correctly answering all questions. Total marks 80.

Lodging of Paper with Baillieu Library:
This paper is to be lodged with the Baillieu library.

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Beginning of exam paper

- 1 (a) Define the notion of a domain in the complex plane \mathbf{C} and decide whether the following subsets of \mathbf{C} are domains:

(1) $\{z : |z| \leq 3\}$, (2) the cut plane \mathbf{C}_π .

Indicate the reasons for your answers.

[6]

- (b) State Cauchy Convergence Criterion. Using it or otherwise, show that a series

$$\sum_{n=0}^{\infty} a_n$$

of complex numbers converges if

$$\sum_{n=0}^{\infty} |a_n|$$

converges.

[7]

- 2 (a) State and derive the Cauchy-Riemann equations.

[7]

- (b) Show that if $f(z)$ and $f(\bar{z})$ are both analytic at z_0 , then $f'(z_0) = 0$.

[6]

- 3 (a) Determine the radius of convergence of the following power series:

$$(1) \sum_{n=0}^{\infty} z^{2n}, \quad (2) \sum_{n=0}^{\infty} (1+i)^n z^n.$$

[6]

- (b) Show that

$$\operatorname{Log}(1+z) = z - \frac{z^2}{2} + \frac{z^3}{3} - \dots$$

for $|z| < 1$, where $\operatorname{Log}(z)$ denotes the principal value of the logarithmic function. (Hint: Differentiate both sides.)

[7]

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4 (a) State Cauchy's Integral Formula for the n^{th} derivative and evaluate

$$\int_{\gamma} \frac{e^{2z}}{(z-1)^n} dz$$

where γ is the contour given by $\gamma(t) = 1 + e^{it}$, $0 \leq t \leq 2\pi$.

[8]

(b) State and prove Liouville's Theorem on bounded entire functions.

[5]

5 (a) Find the Cauchy-Taylor series of

$$\frac{1}{(z-1)(z-2)}$$

at 0.

[7]

(b) Find the Laurent expansion of

$$\frac{(z+1)}{z(z-4)^3}$$

for $0 < |z| < 4$.

[7]

6 (a) Find all singularities of the function

$$f(z) = \frac{1}{e^z - 1} - \frac{1}{z}$$

and classify them.

[7]

(b) Evaluate

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2 + 1)}$$

[7]

End of exam paper