

DEPARTMENT OF MATHEMATICS AND STATISTICS

620–161 INTRODUCTORY MATHEMATICS

Example Sheet 1 – Linear Equations

1. For each of the following linear systems, draw a careful sketch graph and write down the solution from the graph.

$$\begin{array}{llll} \text{(a)} & x - y = 2 & \text{(b)} & 3x - 2y = 5 \\ & x + y = 4 & & 2x - y = 4 \end{array} \quad \begin{array}{ll} \text{(c)} & 3x - 2y = 5 \\ & -6x + 4y = 0 \end{array} \quad \begin{array}{l} \text{(d)} & 3x - 2y = 5 \\ & -6x + 4y = -10 . \end{array}$$

2. For each system in question 1 above, write down an augmented matrix, reduce that matrix, and hence solve the system.

3. Solve graphically the following overdetermined systems :

$$\begin{array}{ll} \text{(a)} & x - 3y = -5 \\ & 2x + y = 4 \\ & x + 3y = 7 \end{array} \quad \begin{array}{l} \text{(b)} & 2x + 3y = 1 \\ & x + 5y = -3 \\ & x - y = 17 . \end{array}$$

4. In each of the following questions, write down an augmented matrix corresponding to the given linear system. Reduce the matrix, showing carefully the row operations you have made. Hence solve each system, or show that there is no solution.

$$\begin{array}{ll} \text{(a)} & x + y + 2z = 23 \\ & y + z = 12 \\ & y - z = -2 \end{array} \quad \begin{array}{l} \text{(b)} & 2x_1 - 3x_2 + 5x_3 = 10 \\ & 4x_1 - 7x_2 - 2x_3 = -5 \\ & 2x_1 - 4x_2 + 25x_3 = 49 \end{array}$$
$$\begin{array}{ll} \text{(c)} & 3x_1 - 4x_2 + x_3 = 2 \\ & -5x_1 + 6x_2 + 10x_3 = 7 \\ & 7x_1 - 10x_2 + 5x_3 = 6 \end{array} \quad \begin{array}{l} \text{(d)} & x_1 + 2x_2 - x_3 = -1 \\ & 2x_1 + 7x_2 - 5x_3 = 3 \\ & 7x_1 + 6x_2 + x_3 = -6 . \end{array}$$

5. Using Gauss-Jordan elimination, find the solutions (if any) of each of the following overdetermined systems :

$$\begin{array}{ll} \text{(a)} & x_1 - 3x_2 + 4x_3 = 13 \\ & 2x_1 + x_2 + x_3 = 5 \\ & 4x_1 + 4x_2 - x_3 = 3 \\ & 3x_1 - 2x_2 + 4x_3 = 17 \end{array} \quad \begin{array}{l} \text{(b)} & 3x_1 - 4x_2 + x_3 = 20 \\ & x_2 + x_3 = 5 \\ & 2x_1 - 3x_2 + 2x_3 = 16 \\ & 5x_1 - 2x_2 + 3x_3 = 7 . \end{array}$$

6. Use Gauss-Jordan elimination, find all solutions of the following underdetermined systems :

$$\begin{array}{ll}
 \text{(a)} & \begin{array}{l} x_1 - 4x_2 + 2x_3 = 10 \\ 3x_1 - 8x_2 + 4x_3 = 8 \end{array} \\
 \text{(b)} & \begin{array}{l} x_1 + x_3 = 6 \\ x_2 + x_3 + x_4 = 2 \\ x_3 + 2x_4 = 6 . \end{array}
 \end{array}$$

7. Using Gauss-Jordan elimination, solve each of the following linear systems :

$$\begin{array}{ll}
 \text{(a)} & \begin{array}{l} 4x_1 + 3x_2 - 2x_3 = -1 \\ x_1 + 2x_2 - 3x_3 + x_4 = 0 \\ 2x_1 - x_2 + x_3 - 3x_4 = 5 \\ x_1 - 3x_2 + x_3 - 2x_4 = -8 \end{array} \\
 \text{(b)} & \begin{array}{l} x_1 - 2x_2 - 3x_3 + x_4 = 11 \\ 2x_1 + x_2 - x_3 - x_4 = -4 \\ 4x_1 - x_2 - x_3 + 2x_4 = 27 \\ x_1 + x_2 - 3x_3 + 4x_4 = 26 . \end{array}
 \end{array}$$

8. The following augmented matrices arise when solving various systems of equations. Find the solutions implied by these augmented matrices or give reasons why there is no solution. The variables corresponding to the columns on the left of the augmentation in the matrices below are x_1, x_2, x_3, x_4 respectively as required.

$$\begin{array}{lll}
 \text{(a)} & \left[\begin{array}{ccc|c} 1 & 0 & & 1 \\ 0 & 1 & & 2 \\ 0 & 0 & & 3 \end{array} \right] & \text{(b)} & \left[\begin{array}{ccc|c} 1 & 0 & 3 & 4 \\ 0 & 1 & 2 & 5 \end{array} \right] & \text{(c)} & \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & 6 \end{array} \right] \\
 \text{(d)} & \left[\begin{array}{ccc|c} 1 & 0 & & 1 \\ 0 & 1 & & 2 \\ 0 & 0 & & 0 \end{array} \right] & \text{(e)} & \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{array} \right] & \text{(f)} & \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 2 \\ 0 & 1 & 0 & 2 & 3 \\ 0 & 0 & 1 & 4 & -1 \end{array} \right]
 \end{array}$$

9. A traveler who just returned from Europe spent:

for housing: \$30/day in England, \$20/day in France, \$20/day in Spain

for food: \$20/day in England, \$30/day in France, \$20/day in Spain

for incidental expenses: \$10/day in each country.

The traveler's records of the trip indicate a total of \$340 spent for housing, \$320 for food, \$140 for incidental expenses while travelling in these countries. Calculate the number of days spent in each country or show that the records must be incorrect.

Suggested extra practice problems (Barnett & Ziegler)

Exercises 4-1 Q 5, 7, 39.

Exercises 4-2 Q 45, 47, 49.

Exercises 4-3 Q 1, 3, 5, 7, 25, 27, 29, 31.

Other Exercises from Barnett & Ziegler Sections 4-1, 4-2, 4-3 may be both relevant and interesting.