620-161 SOLUTIONS TO HOMEWORK SHEET 1
Semester 1 2007

Out of 12 - Parts (a), (b) and (c) each out of 4.

(a) Writing the system in augmented matrix form and using Gauss-Jordan elimination we have:

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 2 & -1 & 0 & 3 \\ 1 & -3 & 1 & 12 \end{array}\right) \begin{array}{l} R_2 - 2R_1 \to R_2 \\ R_3 - R_1 \to R_3 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & -3 & -4 & -3 \\ 0 & -4 & -1 & 9 \end{array}\right) \; -\tfrac{1}{3}R_2 \to R_2$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 1 & \frac{4}{3} & 1 \\ 0 & -4 & -1 & 9 \end{array}\right) \begin{array}{l} R_1 - R_2 \to R_1 \\ \\ R_3 + 4R_2 \to R_3 \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & 2 \\ 0 & 1 & \frac{4}{3} & 1 \\ 0 & 0 & \frac{13}{3} & 13 \end{array}\right) \; \tfrac{3}{13}R_3 \to R_3$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & 2 \\ 0 & 1 & \frac{4}{3} & 1 \\ 0 & 0 & 1 & 3 \end{array}\right) \begin{array}{l} R_1 - \frac{2}{3}R_3 \to R_1 \\ R_2 - \frac{4}{3}R_3 \to R_2 \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

Therefore the system has the unique solution

$$(x, y, z) = (0, -3, 3)\ .$$

CHECK:
Substituting into the original equations we have:

$$\begin{aligned} (0) + (-3) + 2(3) &= 3 \quad ✓ \\ 2(0) - (-3) \qquad &= 3 \quad ✓ \\ (0) - 3(-3) + (3) &= 12 \quad ✓ \end{aligned}$$

(b) Applying Gauss-Jordan elimination to this system yields:

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 2 & -1 & 0 & 3 \\ 1 & -2 & -2 & 12 \end{array}\right) \begin{array}{l} R_2 - 2R_1 \to R_2 \\ R_3 - R_1 \to R_3 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & -3 & -4 & -3 \\ 0 & -3 & -4 & 9 \end{array}\right) \; R_3 - R_2 \to R_3$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & -3 & -4 & -3 \\ 0 & 0 & 0 & 12 \end{array}\right)$$

The last row says "0=12" so the system is inconsistent and there are no real solutions.

(c) Gauss-Jordan elimination on this system gives:

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 2 & -1 & 0 & 3 \\ 2 & 2 & 4 & 6 \end{array}\right) \begin{array}{l} R_2 - 2R_1 \rightarrow R_2 \\ R_3 - 2R_1 \rightarrow R_3 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & -3 & -4 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right) -\tfrac{1}{3}R_2 \rightarrow R_2$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 1 & \frac{4}{3} & 1 \\ 0 & 0 & 0 & 0 \end{array}\right) R_1 - R_2 \rightarrow R_1$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & 2 \\ 0 & 1 & \frac{4}{3} & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

From the final matrix we read the equations:

$$x + \frac{2}{3}z = 2 \qquad \text{and} \qquad y + \frac{4}{3}z = 1 \ .$$

Since there is no row leader in the z column we let $z = t$, $t \in \mathbb{R}$, be a parameter. Solving for x and y in terms of t then yields:

$$x = 2 - \frac{2}{3}t \qquad \text{and} \qquad y = 1 - \frac{4}{3}t \ ,$$

so that the solution is:

$$(x, y, z) = \left(2 - \frac{2}{3}t, 1 - \frac{4}{3}t, t\right) , \qquad t \in \mathbb{R} \ .$$

CHECK:
We can again check our solution by substitution into the original equations:

$$\begin{aligned} \left(2 - \frac{2}{3}t\right) + \left(1 - \frac{4}{3}t\right) + 2(t) &= 3 \quad \checkmark \\ 2\left(2 - \frac{2}{3}t\right) - \left(1 - \frac{4}{3}t\right) &= 3 \quad \checkmark \\ 2\left(2 - \frac{2}{3}t\right) + 2\left(1 - \frac{4}{3}t\right) + 4(t) &= 6 \quad \checkmark \end{aligned}$$