620-161 SOLUTIONS TO HOMEWORK SHEET 2
Semester 1 2007

Out of 8 - Parts (a) and (b) each out of 4.

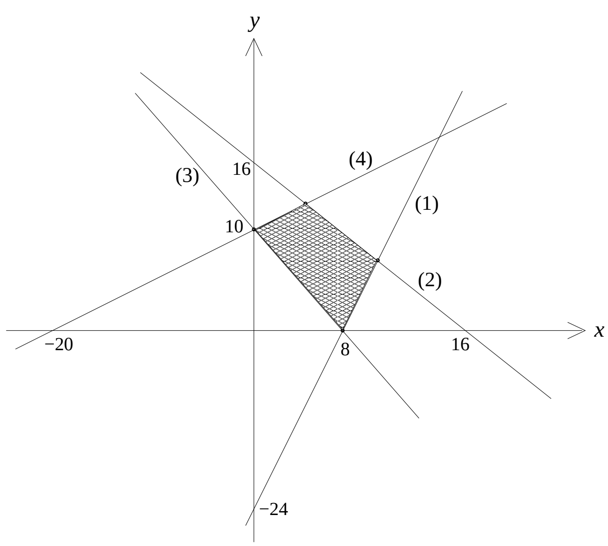
(a) The feasible region is the same for parts (i) and (ii) and is given by the inequalities:

$$\begin{aligned} 3x - y &\leq 24 \\ x + y &\leq 16 \\ 5x + 4y &\geq 40 \\ -x + 2y &\leq 20 \\ x \geq 0, y &\geq 0 \end{aligned}$$

Consider the bounding lines:

(1) :	$3x - y = 24$	When $x = 0,\ y = -24.$	When $y = 0,\ x = 8.$
(2) :	$x + y = 16$	When $x = 0,\ y = 16.$	When $y = 0,\ x = 16.$
(3) :	$5x + 4y = 40$	When $x = 0,\ y = 10.$	When $y = 0,\ x = 8.$
(4) :	$-x + 2y = 20$	When $x = 0,\ y = 10.$	When $y = 0,\ x = -20.$

After testing whether the origin lies in each half-plane, we find that the feasible region is the shaded region below:



We need to determine the two corner points not on the axes.
Solving simultaneously, for the intersection of lines (1) and (2) we have:

$$(1) + (2): \qquad 4x = 40 \quad \Rightarrow \quad x = 10 \quad \Rightarrow \quad y = 16 - x = 16 - 10 = 6\ , \quad \text{i.e. } (10, 6)\ .$$

And for the intersection of lines (2) and (4) we have:

$$(2) + (4): \qquad 3y = 36 \quad \Rightarrow \quad y = 12 \quad \Rightarrow \quad x = 16 - y = 16 - 12 = 4\ , \quad \text{i.e. } (4, 12)\ .$$

(i) We want to maximise $P = 2x + y$ on the above feasible region. The optimal value of P will occur at a corner point. We have

$$\begin{aligned}
P(8,0) &= 2(8) + 0 = 16 \\
P(0,10) &= 2(0) + 10 = 10 \\
P(10,6) &= 2(10) + 6 = 26 \\
P(4,12) &= 2(4) + 12 = 20
\end{aligned}$$

So the optimal solution is $(x, y) = (10, 6)$ giving the optimal value $P = 26$.

CHECK:
Substituting into the constraints, we have:

$$\begin{aligned}
3(10) - 6 = 24 &\leq 24 \quad \checkmark \\
10 + 6 = 16 &\leq 16 \quad \checkmark \\
5(10) + 4(6) = 74 &\geq 40 \quad \checkmark \\
-(10) + 2(6) = 2 &\leq 20 \quad \checkmark \\
\text{and} \qquad 10,\ 6 &\geq 0 \quad \checkmark
\end{aligned}$$

(ii) We now want to maximise $P = 3x + 4y$. We have

$$\begin{aligned}
P(8,0) &= 3(8) + 4(0) = 24 \\
P(0,10) &= 3(0) + 4(10) = 40 \\
P(10,6) &= 3(10) + 4(6) = 54 \\
P(4,12) &= 3(4) + 4(12) = 60
\end{aligned}$$

So the optimal solution is $(x, y) = (4, 12)$ giving the optimal value $P = 60$.

CHECK:
Substituting into the constraints, we have:

$$\begin{aligned}
3(4) - 12 = 0 &\leq 24 \quad \checkmark \\
4 + 12 = 16 &\leq 16 \quad \checkmark \\
5(4) + 4(12) = 68 &\geq 40 \quad \checkmark \\
-(4) + 2(12) = 20 &\leq 20 \quad \checkmark \\
\text{and} \qquad 4,\ 12 &\geq 0 \quad \checkmark
\end{aligned}$$

(b) Let

c be the number of batches of capsules manufactured (3000 per batch),

and t be the number of batches of tablets manufactured (3000 per batch).

So c and t must be nonnegative integers.
From the worded problem we also observe the following:

time constraint:	$3\frac{1}{2}\,c + 2\frac{1}{2}\,t \leq 25$
resource constraints:	$c \leq 5$ and $t \leq 7$
objective function (profit (\$)):	$P = 175c + 150t$

In summary, the linear programming problem is:

$$\begin{aligned} \text{Maximise} \quad & P = 175c + 150t \\ \text{subject to} \quad & 3\frac{1}{2}\,c + 2\frac{1}{2}\,t \leq 25 \\ & c \leq 5 \\ & t \leq 7 \\ & c, t \geq 0 \end{aligned}$$