620-161 SOLUTIONS TO HOMEWORK SHEET 3
Semester 1 2007

Out of 4.

$$\begin{aligned} \text{Maximise} \quad & P = 2x_1 + 3x_2 + x_3 \\ \text{subject to} \quad & 2x_1 + x_2 + x_3 \le 15 \\ & 3x_1 - 2x_2 + x_3 \le 11 \\ & x_1 + 2x_2 + 4x_3 \le 10 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

First rewrite, introducing slack variables:

$$\begin{aligned} 2x_1 + x_2 + x_3 + s_1 \qquad\qquad\qquad &= 15 \\ 3x_1 - 2x_2 + x_3 \qquad + s_2 \qquad\qquad &= 11 \\ x_1 + 2x_2 + 4x_3 \qquad\qquad + s_3 \qquad &= 10 \\ -2x_1 - 3x_2 - x_3 \qquad\qquad\qquad + P &= 0 \qquad \text{(objective function)} \\ x_1, x_2, x_3, s_1, s_2, s_3 &\ge 0 \end{aligned}$$

Then rewrite in a simplex tableau:

BV	x_1	x_2	x_3	s_1	s_2	s_3	P	RHS		ratio test:	
s_1	2	1	1	1	0	0	0	15		$\frac{15}{1} = 15$	$R_1 - R_3' \to R_1$
s_2	3	-2	1	0	1	0	0	11		$-$	$R_2 + 2R_3' \to R_2$
s_3	1	$\boxed{2}$	4	0	0	1	0	10	$\longleftarrow$	$\frac{10}{2} = 5$	$\frac{1}{2}R_3 \to R_3'$
P	-2	-3	-1	0	0	0	1	0			$R_4 + 3R_3' \to R_4$
		$\uparrow$									

BV	x_1	x_2	x_3	s_1	s_2	s_3	P	RHS		ratio test:	
s_1	$\frac{3}{2}$	0	-1	1	0	$-\frac{1}{2}$	0	10		$\frac{10}{3/2} = 6\frac{2}{3}$	$R_1 - \frac{3}{2}R_2' \to R_1$
s_2	$\boxed{4}$	0	5	0	1	1	0	21	$\longleftarrow$	$\frac{21}{4} = 5\frac{1}{4}$	$\frac{1}{4}R_2 \to R_2'$
x_2	$\frac{1}{2}$	1	2	0	0	$\frac{1}{2}$	0	5		$\frac{5}{1/2} = 10$	$R_3 - \frac{1}{2}R_2' \to R_3$
P	$-\frac{1}{2}$	0	5	0	0	$\frac{3}{2}$	1	15			$R_4 + \frac{1}{2}R_2' \to R_4$
	$\uparrow$										

BV	x_1	x_2	x_3	s_1	s_2	s_3	P	RHS
s_1	0	0	$-\frac{23}{8}$	1	$-\frac{3}{8}$	$-\frac{7}{8}$	0	$\frac{17}{8}$
x_1	1	0	$\frac{5}{4}$	0	$\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{21}{4}$
x_2	0	1	$\frac{11}{8}$	0	$-\frac{1}{8}$	$\frac{3}{8}$	0	$\frac{19}{8}$
P	0	0	$\frac{45}{8}$	0	$\frac{1}{8}$	$\frac{13}{8}$	1	$17\frac{5}{8}$

Since there are no more negative entries in the bottom row we are done.

Therefore the optimal solution is obtained when:

$$s_1 = \frac{17}{8}, \quad x_1 = \frac{21}{4}, \quad x_2 = \frac{19}{8}, \quad x_3, s_2, s_3 = 0 \ ,$$

$$\text{ie.} \qquad (x_1, x_2, x_3) = \left(\frac{21}{4}, \frac{19}{8}, 0\right) \ ,$$

giving optimal value, max $P = 17\frac{5}{8}$.

CHECK:
We have

$$\text{Max } P = 2\left(\frac{21}{4}\right) + 3\left(\frac{19}{8}\right) + (0) = \frac{84+57}{8} = \frac{141}{8} = 17\frac{5}{8} \quad \checkmark$$

and substituting into the constraints:

$$\begin{aligned}
2\left(\frac{21}{4}\right) + \left(\frac{19}{8}\right) + (0) &= \frac{84+19}{8} = \frac{103}{8} = 12\frac{7}{8} \le 15 \quad \checkmark \\
3\left(\frac{21}{4}\right) - 2\left(\frac{19}{8}\right) + (0) &= \frac{126-38}{8} = \frac{88}{8} = 11 \le 11 \quad \checkmark \\
\left(\frac{21}{4}\right) + 2\left(\frac{19}{8}\right) + 4(0) &= \frac{42+38}{8} = \frac{80}{8} = 10 \le 10 \quad \checkmark \\
\text{and} \qquad & \frac{21}{4}, \frac{19}{8}, 0 \ge 0 \quad \checkmark
\end{aligned}$$