

620-161 SOLUTIONS TO HOMEWORK SHEET 5
Semester 1 2007

Out of 8 - Parts (a) and (b) each out of 4.

(a) (i)

$$\begin{aligned} f(x) &= \log_e[x(x^2 + 1)^2] \\ \Rightarrow f'(x) &= \frac{1}{x(x^2 + 1)^2} \cdot \frac{d}{dx}[x(x^2 + 1)^2] \\ &= \frac{1}{x(x^2 + 1)^2} \cdot ((x^2 + 1)^2 + x \cdot 2(x^2 + 1) \cdot 2x) \\ &= \frac{5x^2 + 1}{x(x^2 + 1)} \\ \text{or} &= \frac{1}{x} + \frac{4x}{x^2 + 1} \end{aligned}$$

(ii)

$$\begin{aligned} f(x) &= e^{2x}(x^3 + 3x + 1) \\ \Rightarrow f'(x) &= 2e^{2x}(x^3 + 3x + 1) + e^{2x}(3x^2 + 3) \\ &= e^{2x}(2x^3 + 3x^2 + 6x + 5) \end{aligned}$$

(iii)

$$\begin{aligned} f(x) &= (1 + x^3)^{\frac{1}{2}} \\ \Rightarrow f'(x) &= \frac{1}{2}(1 + x^3)^{-\frac{1}{2}} \cdot 3x^2 \\ &= \frac{3x^2}{2\sqrt{1 + x^3}} \end{aligned}$$

(b) Let L, W, P (in inches) denote the length, width and perimeter of the rectangle, respectively. Let A (in square inches) be the area of the rectangle. Let t denote time (in seconds). Then

$$P = 2L + 2W, \quad A = LW,$$

and we are told

$$\frac{dL}{dt} = 3.$$

(i) The given condition says

$$A = LW = 16 \quad \Rightarrow \quad W = \frac{16}{L},$$

and we want to find $\frac{dP}{dt}$. We have

$$P = 2L + 2W = 2L + 2 \cdot \frac{16}{L} = 2L + \frac{32}{L},$$

so by the chain rule

$$\frac{dP}{dt} = \frac{dP}{dL} \frac{dL}{dt} = \left(2 - \frac{32}{L^2}\right) \cdot 3 = 6 - \frac{96}{L^2}.$$

When $L = 5$ we have

$$\frac{dP}{dt} = 6 - \frac{96}{25} = \frac{54}{25},$$

so the perimeter is changing at a rate of $\frac{54}{25} = 2.16$ inches per second when $L = 5$.

(ii) The given condition says

$$P = 2L + 2W = 16 \quad \Rightarrow \quad W = \frac{1}{2}(16 - 2L),$$

and we want to find $\frac{dA}{dt}$. We have

$$A = LW = L \cdot \frac{1}{2}(16 - 2L) = 8L - L^2,$$

so by the chain rule

$$\frac{dA}{dt} = \frac{dA}{dL} \frac{dL}{dt} = (8 - 2L) \cdot 3 = 24 - 6L.$$

When the rectangle is a square, the length and width are equal at 4 inches, so

$$\frac{dA}{dt} = 24 - 6(4) = 0,$$

and the area is not changing (or changing at a rate of 0 inches per second) when the rectangle is a square.