

620-161 SOLUTIONS TO HOMEWORK SHEET 8
Semester 1 2007

Out of 8 - Parts (a) and (b) each out of 4.

(a) We have $f(x, y) = x^4y^3 + (x - 1) \sin y$.

(i) The partial derivatives are:

$$\begin{aligned}\frac{\partial f}{\partial x} &= 4x^3y^3 + \sin y \\ \frac{\partial f}{\partial y} &= 3x^4y^2 + (x - 1) \cos y \\ \frac{\partial^2 f}{\partial x^2} &= 12x^2y^3 \\ \frac{\partial^2 f}{\partial y^2} &= 6x^4y - (x - 1) \sin y \\ \frac{\partial^2 f}{\partial x \partial y} &= \frac{\partial^2 f}{\partial y \partial x} = 12x^3y^2 + \cos y\end{aligned}$$

(ii) From (i) above we have

$$\begin{aligned}\frac{\partial f}{\partial x}(1, 1) &= 4 + \sin 1 \\ \text{and } \frac{\partial f}{\partial y}(1, 1) &= 3 + 0 = 3.\end{aligned}$$

So the tangent plane of the surface $z = f(x, y)$ at the point $(1, 1, 1)$ is

$$\begin{aligned}z - 1 &= \frac{\partial f}{\partial x}(1, 1)(x - 1) + \frac{\partial f}{\partial y}(1, 1)(y - 1) \\ &= (4 + \sin 1)(x - 1) + 3(y - 1).\end{aligned}$$

(b) We have the estimated profit function given (in dollars) by

$$P(s, g) = -50000 + 300s + 1000g - 3s^2 - g^2 + sg.$$

(i) If $s = 100$ and $g = 500$, then the estimated profit is

$$\begin{aligned}P(100, 500) &= -50000 + 300(100) + 1000(500) - 3(100)^2 - (500)^2 + (100)(500) \\ &= 250000\end{aligned}$$

as required.

(ii) We have $P_s(s, g) = 300 - 6s + g$ and $P_g(s, g) = 1000 - 2g + s$. Hence

$$\begin{aligned}P_s(100, 500) &= 300 - 6 \cdot 100 + 500 = 200 \\ \text{and } P_g(100, 500) &= 1000 - 2 \cdot 500 + 100 = 100.\end{aligned}$$

(iii) A \$1 increase in the price of both medallions means $\Delta s = \Delta g = 1$, so the approximate affect of this on the estimated profit is

$$\begin{aligned}\Delta P &\approx P_s(100, 500)\Delta s + P_g(100, 500)\Delta g \\ &= 200 \cdot 1 + 100 \cdot 1 \\ &= 300 .\end{aligned}$$

That is, the profit increases by approximately \$300.