620-161 Introductory Mathematics

Problems Set 3

1. (a) Use Gauss-Jordan elimination to solve the following system of linear equations. At each step indicate clearly the row operation that you perform.

$$\begin{aligned} -x + 3y + 2z &= 8 \\ 2x - y + z &= -1 \\ 3x + y - z &= 16. \end{aligned}$$

(b) Write down the solutions implied by the following matrices:

(i)

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

(ii)

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

(iii)

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

(c) Which (if any) of the solutions that you found in (b) are solutions to the following system?

$$\begin{aligned} 3x + 2y + 3z &= 0 \\ 5x - 2y + 5z &= 16 \\ x + 3y + z &= -7? \end{aligned}$$

2. (a) Use the Graphical method to solve the following linear programming problems (note that the feasible region is the same in both cases):

(i)

$$\begin{aligned} \text{Maximize P} = {} & 4x + 7y \\ \text{subject to} \quad & 3x + y \leq 27 \\ & 2x + 3y \geq 18 \\ & 2y - x \leq 8 \\ & x,\ y \geq 0. \end{aligned}$$

(ii)

$$\begin{aligned} \text{Minimize P} = {} & 4x + 7y \\ \text{subject to} \quad & 3x + y \leq 27 \\ & 2x + 3y \geq 18 \\ & 2y - x \leq 8 \\ & x,\ y \geq 0. \end{aligned}$$

Remember to clearly label all intersection points and the feasible region.

(b) Write down a linear programming formulation for the problem below. You should explain your notation clearly. **DO NOT ATTEMPT TO SOLVE THE PROBLEM**.

A new fast food chain plans to open several new restaurants. Each restaurant will have eat-in facilities and some will have drive-through service.

A restaurant without drive-through service will cost \$200,000 to build and will employ 15 people. A restaurant with drive-through service will cost \$150,000 to build and employ 12 people. The expected annual revenue for a drive-through restaurant is \$500,000 whilst a restaurant which has only eat-in facilities can expect \$400,000 annual revenue.

The fast food chain can spend at most \$1,400,000 on its construction works and its labour market is at most 100 people. How many of each type of restaurant should the company build to maximize its revenue?

3. (a) Use the Simplex Method to solve the following linear programming problem. At each step you should circle the pivot element and specify which row operations you are using.

$$\begin{aligned} \text{Maximize P} = {} & 4x_1 + x_2 + 3x_3 \\ \text{subject to} \quad & 2x_1 + x_2 - x_3 \leq 6 \\ & 6x_1 + 3x_2 + 3x_3 \leq 3 \\ & x_1,\ x_2,\ x_3 \geq 0. \end{aligned}$$

(b) (i) Write the dual of the following linear programming problem:

$$\begin{aligned} \text{Minimize C} = \quad & 6y_1 + 3y_2 \\ \text{subject to} \quad & 2y_1 + 6y_2 \geq 4 \\ & y_1 + 3y_2 \geq 1 \\ & -y_1 + 3y_2 \geq 3 \\ & y_1,\ y_2 \geq 0. \end{aligned}$$

(ii) Is $y^* = (1,2)$, $z^* = 12$ the optimal solution to 3(b)? Justify your answer.

4. (a) Find the derivatives of the following functions:

(i) $f(x) = (\log_e(5x) + 3x^2)\,(x^2 + 2x + 3)^{\frac{1}{2}}$.

(ii) $f(x) = \cos^2(x^3 + 3x^2 + 2)$.

(b) A spectator standing at a distance of 3,000 m from a launch site, is observing a rocket launch. The rocket is launched vertically and is rising at a rate of 300 m/sec.

When its altitude is 4,000 m how fast is the distance between the rocket and the spectator changing at that instant?

5. (a) Use implicit differentiation to find $\frac{dy}{dx}$ at the point $(2, 1)$ in the case where

$$\log_e(y) = x^3y + y.$$

(b) (i) Construct the Taylor polynomial of order 3 about $x = 0$ for the function

$$f(x) = \cos(3x) + x^2.$$

(ii) Write down an expression for the error $E_3(x)$ if this polynomial is used to approximate $f(x)$ at $x = \frac{1}{3}$.

(iii) Compute an upper bound for this error.

6. (a) Let $f(x) = 2x^3 - 3x^2 - 36x + 20$.

(i) Find all stationary points of $f(x)$ in $(-\infty, \infty)$.

(ii) Determine the nature (local minimum, local maximum, or neither) of each of the stationary points that you found in (i).

(iii) Find the absolute maximum value and absolute minimum value of $f(x)$ for $-1 \leq x \leq 4$.

(b) Show that the rectangle having a given area (say, A m^2 for some constant $A > 0$) is a square when the perimeter has minimum length.

7. (a) Let $z = \log_e(x^2 + y^2) + 2xy - 1$. Find

$$\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y^2} \text{ and } \frac{\partial^2 z}{\partial x \partial y}.$$

Show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0.$$

(b) The radius r and height h of a right circular cylinder are found by measurement to be $r = 6.01$ feet, $h = 9.02$ feet. Find the approximate error in the volume of the cylinder from these measurements if the actual measurements are $r = 6$ feet and $h = 9$ feet. (The volume of a right circular cylinder with radius r and height h is $V = \pi r^2 h$.)

8. (a) Find all stationary points of the function

$$f(x, y) = x^3 + y^3 - 6xy + 1$$

and determine (if possible) the nature (local minimum, local maximum, saddle point) of each one by means of the Second Derivative Test.

(b) Using the Method of Lagrange Multipliers, find the maximum area of a rectangle that can be inscribed in the circle $x^2 + y^2 = 1$. See Figure 1 and note that the coordinates of the four corner points of the rectangle, for instance point A, should satisfy the equation $x^2 + y^2 = 1$.

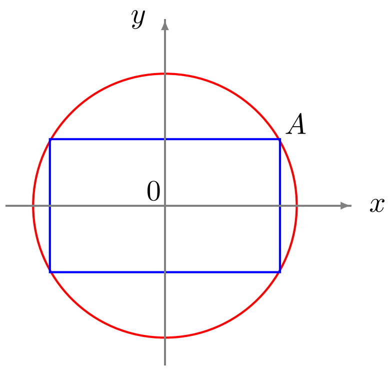

Figure 1:

9. (a) Let $z_1 = 2 - 3i$ and $z_2 = 1 + i$. Simplify

(i) $z_1 + z_2$;

(ii) $2z_1 - 5z_2$;

(iii) $z_1 z_2^2$;

(iv) $\frac{z_1}{z_2}$.

You are required to express your answers in the form $a + ib$, where a, b are real numbers.

(b) Plot the complex numbers you have calculated in (i)-(iv) on one Argand diagram.

(c) Find all solutions for each of the following equations:

(i) $z^2 + 4z + 7 = 0$;

(ii) $4z^3 - 4z^2 + 3z = 0$.

10. (a) Consider

$$\frac{dy}{dx} = \frac{(x+1)y}{x^2}.$$

(i) Verify that $y = Cxe^{-\frac{1}{x}}$ is the general solution of this differential equation, where C is an arbitrary constant.

(ii) Find the particular solution of this equation which satisfies $y(1) = 2$.

(b) Find the general solution of the following differential equations:

(i) $\frac{dy}{dx} = -2e^x + 5x^4 - x$.

(ii) $\frac{dy}{dx} = y + 1, \; y > 0$.

Solutions

1. (a)

$$\left[\begin{array}{rrr|r} -1 & 3 & 2 & 8 \\ 2 & -1 & 1 & -1 \\ 3 & 1 & -1 & 16 \end{array}\right]$$

$$\sim \left[\begin{array}{rrr|r} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & -5 & 10 \end{array}\right]$$

$$\sim \left[\begin{array}{rrr|r} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

So $x = 3,\ y = 5,\ z = -2$.
Checking the solutions in each of the equations we have

(i) $-3 + 3(5) + 2(-2) = 8$
(ii) $2(3) - 5 - 2 = -1$
(iii) $3(3) + 5 - (-2) = 16$

as required.

(b) (i) $x = 0,\ y = -3,\ z = 2$.
(ii) No real solutions since $0.x + 0.y + 0.z \neq 2$ for any $x,\ y,\ z$.
(iii) Put $z = t \in \mathbb{R}$. Then $y = -3$ and $x = 2 - t$

(c) Checking the solutions in each of the equations we have

(i) $3(0) + 2(-3) + 3(2) = 0$
(ii) $5(0) - 2(-3) + 5(2) = 16$
(iii) $0 + 3(-3) + 2 = -7$.

(iii) (i) $3(2 - t) + 2(-3) + 3(t) = 0$
(ii) $5(2 - t) - 2(-3) + 5(t) = 16$
(iii) $2 - t + 3(-3) + t = -7$.

Otherwise observe that t=2 gives the same solution as (i).

2. (a) Diagram for problem.

$$P(9,0) = 36 \quad \text{(min)}$$
$$P(46/7, 51/7) = 77\ 2/7 \quad \text{(max)}$$
$$P(12/7, 34/7) = 40\ 3/7$$

(b) Let x represent the number of eat-in only restaurants and y be the number of restaurants with drive-through. The company wants to wants to maximize its revenue R according to:

$$\begin{aligned} \text{Maximize R} = \ & 400,000x + 500,000y \\ \text{subject to} \quad & 200,000x + 150,000y \leq 1,400,000 \\ & 15x + 12y \leq 100 \\ & x,\ y \geq 0. \end{aligned}$$

3. (a) Writing the equation in standard form

$$\begin{aligned} \text{Maximize P} - 4x_1 + x_2 + 3x_3 &= 0 \\ \text{subject to} \quad 2x_1 + x_2 - x_3 + s_1 &= 6 \\ 6x_1 + 3x_2 + 3x_3 + s_2 &= 3 \\ x_1,\ x_2,\ x_3,\ s_1,\ s_2 &\geq 0. \end{aligned}$$

BV	x_1	x_2	x_3	s_1	s_2	RHS
s_1	2	1	-1	1	0	6
s_2	6	3	3	0	1	3
P	-4	-1	-3	0	0	0

BV	x_1	x_2	x_3	s_1	s_2	RHS
s_1	0	0	-2	1	$-1/3$	5
x_1	1	1/2	1/2	0	1/6	1/2
P	0	1	-1	0	2/3	2

BV	x_1	x_2	x_3	s_1	s_2	RHS
s_1	4	2	0	1	1/3	7
x_3	2	1	1	0	1/3	1
P	2	2	0	0	1	3

So $x^* = (0,0,1),\ P^* = 3.$

(b) (i)

$$\begin{aligned} \text{Maximize P} = {} & 4x_1 + x_2 + 3x_3 \\ \text{subject to} \quad & 2x_1 + x_2 - x_3 \leq 6 \\ & 6x_1 + 3x_2 + 3x_3 \leq 3 \\ & x_1,\ x_2,\ x_3 \geq 0. \end{aligned}$$

(ii) No it isn't. From the previous tableau we see that $y^* = (0, 1)$ $z^* = 3$.

4. (a) (i) If $f(x) = (\log_e(5x) + 3x^2)(x^2 + 2x + 3)^{\frac{1}{2}}$ then
$f'(x) = \left(\frac{1}{x} + 6x\right)((x^2 + 2x + 3)^{\frac{1}{2}} + (\log_e(5x) + 3x^2) \cdot \frac{1}{2}(x^2 + 2x + 3)^{-\frac{1}{2}} \cdot (2x + 2)$.

(ii) If $f(x) = \cos^2(x^3 + 3x^2 + 2)$ then
$f'(x) = -2\cos(x^3 + 3x^2 + 2)\sin(x^3 + 3x^2 + 2)(3x^2 + 6x)$.

(b)

$$x^2 = y^2 + (3,000)^2$$

$$\Longrightarrow 2x\frac{dx}{dt} = 2y\frac{dy}{dt},$$

so

$$x\frac{dx}{dt} = y\frac{dy}{dt}.$$

So when $x = 5,000,\ y = 4,000$ and $\frac{dy}{dt} = 300$ we have

$$\frac{dx}{dt} = \frac{4,000 \times 300}{5,000} = 240 \text{ cm/s}.$$

5. (a)

$$\begin{aligned} \log_e(y) &= x^3y + y \\ \Rightarrow \frac{1}{y}\frac{dy}{dx} &= \frac{dy}{dx} + 3x^2y + x^3\frac{dy}{dx} \\ \Rightarrow \left(\frac{1}{y} - 1 - x^3\right)\frac{dy}{dx} &= 3x^2y \\ \Rightarrow \frac{dy}{dx} &= \frac{3x^2y}{\frac{1}{y} - 1 - x^3}. \end{aligned}$$

So at $(2, 1)$,

$$\frac{dy}{dx} = \frac{12}{-8} = -\frac{3}{2}.$$

(b) (i)

$$
\begin{aligned}
f(x) &= \cos(3x) + x^2 \\
\Rightarrow f'(x) &= -3\sin(3x) + 2x \\
\Rightarrow f''(x) &= -9\cos(3x) + 2 \\
\Rightarrow f''' &= 27\sin(3x) \\
\Rightarrow f^{(iv)} &= 81\cos(3x)
\end{aligned}
$$

$$
\begin{aligned}
f(0) &= 1 \\
\Rightarrow f'(0) &= 0 \\
\Rightarrow f''(0) &= -7 \\
\Rightarrow f'''(0) &= 0 \\
\Rightarrow f^{(iv)}(c) &= 81\cos(3c).
\end{aligned}
$$

So

$$
\begin{aligned}
P_3(x) &= f(0) + f'(0)x + f''(0)\frac{x^2}{2} + f'''(0)\frac{x^3}{3!} \\
&= 1 - \frac{7x^2}{2!}.
\end{aligned}
$$

(ii) According to Taylor's Theorem the error term is as follows:

$$
\begin{aligned}
E_3(x) &= \frac{f^{(iv)}(c)}{4!}x^4 \\
&= \frac{81\cos(3c)}{4!}x^4 = \frac{27\cos(3c)}{8}x^4
\end{aligned}
$$

for some $c \in [0, 1/3]$.

(iii) As the cosine function is *decreasing* on the interval $[0, 1/3]$, we choose the smallest element of c in this interval, namely $c = 0$, and then we have the upper bound

$$
E_3(x) \leq \frac{27\cos(0)}{8}x^4 = \frac{27}{8}x^4.
$$

So for $x = 1/3$ we have

$$
E_3(1/3) \leq \frac{27}{8}(1/3)^4 = \frac{1}{24}.
$$

6. (a) (i) We have $f'(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 6(x+2)(x-3)$. Setting $f'(x) = 0$ we get two stationary points, namely $x = -2$ and $x = 3$.

(ii)

x	$(-\infty, -2)$	-2	$(-2, 3)$	3	$(3, \infty)$
$f'(x)$	$+$	0	$-$	0	$+$
$f(x)$	$\nearrow$		$\searrow$		$\nearrow$

From this diagram it follows that $x = -2$ is a local maximum point and $x = 3$ is a local minimum point over $(-\infty, \infty)$.
Another method: $f''(x) = 12x - 6$, $f''(-2) = -30 < 0$, $f''(3) = 30 > 0$. By the second derivative test, $x = -2$ is a local maximum point and $x = 3$ is a local minimum point.

(iii) The point $x = 3$ is the only stationary point in the interval $[-1, 4]$. Evaluating $f(x)$ at $3, -1, 4$ we have $f(3) = -61, f(-1) = 51, f(4) = -44$. Hence the absolute maximum value of f on $[-1, 4]$ is $f(-1) = 51$ and the absolute minimum value is $f(3) = -61$.

(b) Let x, y be the dimensions of the rectangle. Then $x, y > 0$, $xy = A$, and the perimeter is $P = 2x + 2y$. Thus, $P = 2x + \frac{2A}{x}$ and $dP/dx = 2 - \frac{2A}{x^2} = 2(1 - \frac{\sqrt{A}}{x})(1 + \frac{\sqrt{A}}{x})$. Setting $dP/dx = 0$ we obtain a unique stationary point in $(0, \infty)$, namely $x_0 = \sqrt{A}$. Since $dP/dx < 0$ when $0 < x < \sqrt{A}$, P decreases in the interval $(0, \sqrt{A})$. Similarly, since $dP/dx > 0$ when $\sqrt{A} < x < \infty$, P increases in $(\sqrt{A}, \infty)$. Therefore, $x_0 = \sqrt{A}$ is a global maximum point in $(0, \infty)$. When $x_0 = \sqrt{A}$ we have $y_0 = \frac{A}{x_0} = \sqrt{A}$, and hence $x_0 = y_0 = \sqrt{A}$. That is, the rectangle is a square when P is minimum.

7. (a) We have

$$\frac{\partial z}{\partial x} = \frac{2x}{x^2 + y^2} + 2y, \quad \frac{\partial z}{\partial y} = \frac{2y}{x^2 + y^2} + 2x$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{2(x^2 + y^2) - 4x^2}{(x^2 + y^2)^2} = \frac{2(y^2 - x^2)}{(x^2 + y^2)^2}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{2(x^2 + y^2) - 4y^2}{(x^2 + y^2)^2} = \frac{2(x^2 - y^2)}{(x^2 + y^2)^2}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{0(x^2 + y^2) - 4xy}{(x^2 + y^2)^2} + 2 = -\frac{4xy}{(x^2 + y^2)^2} + 2.$$

From the expressions of $\partial^2 z/\partial x^2$ and $\partial^2 z/\partial y^2$ it is clear that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{2(y^2 - x^2)}{(x^2 + y^2)^2} + \frac{2(x^2 - y^2)}{(x^2 + y^2)^2} = 0.$$

(b) Since $V = \pi r^2 h$, we have $\frac{\partial V}{\partial r} = 2\pi r h$, $\frac{\partial V}{\partial h} = \pi r^2$. Hence

$$\left.\frac{\partial V}{\partial r}\right|_{r=6,h=9} = 108\pi, \quad \left.\frac{\partial V}{\partial h}\right|_{r=6,h=9} = 36\pi.$$

The measurement errors are

$$\Delta r = 0.01, \quad \Delta h = 0.02.$$

Thus, the incremental error in volume is approximately

$$\begin{aligned} \Delta V &= \left.\tfrac{\partial V}{\partial r}\right|_{r=6,h=9} \Delta r + \left.\tfrac{\partial V}{\partial h}\right|_{r=6,h=9} \Delta h \\ &= 108\pi \times 0.01 + 36\pi \times 0.02 \\ &= 1.08\pi + 0.72\pi \\ &= 1.8\pi \text{ (cubic feet).} \end{aligned}$$

8. (a) We have $f_x = 3x^2 - 6y, f_y = 3y^2 - 6x$. Setting $f_x = f_y = 0$ we have

$$x^2 = 2y, \quad y^2 = 2x.$$

Plugging $y = x^2/2$ (which follows from the first equation) into the second equation we then have $x(x^3 - 2^3) = 0$. Hence $x = 0$ or 2, and $y = 0$ or 2 correspondingly. Thus, there are two stationary points: $(0, 0)$ and $(2, 2)$.
We have

$$A = f_{xx} = 6x, \quad B = f_{xy} = -6, \quad C = f_{yy} = 6y.$$

At $(0, 0)$ we have $A = 0, B = -6, C = 0$ and thus $AC - B^2 = -36 < 0$. Hence $(0, 0)$ is a saddle point. At $(2, 2)$ we have $A = 12, B = -6, C = 12$; since $AC - B^2 = 144 - 36 = 108 > 0$ and $A > 0$, $(0, 0)$ is a local minimum point.

(b) Let the 4 corner points of the rectangle be $(-x, -y), (-x, y), (x, y)$ and (x,-y) where $x > 0, y > 0$. The width is then equal to $2x$ and the height to $2y$. The area of the rectangle is given then by $f(x, y) = (2x)(2y) = 4xy$. We are asked to find the maximum value of $f(x, y)$ subject to the constraint $g(x, y) \equiv x^2 + y^2 - 1 = 0$. Let

$$F(x, y, \lambda) = 4xy + \lambda(x^2 + y^2 - 1).$$

Then

$$F_x = 4y + 2\lambda x, \quad F_y = 4x + 2\lambda y, \quad F_\lambda = x^2 + y^2 - 1.$$

Set

$$4y + 2\lambda x = 0$$

$$4x + 2\lambda y = 0$$

$$x^2 + y^2 - 1 = 0.$$

From the first equation we have $y = -\lambda x/2$. Plugging this into the second equation, we get $4x - \lambda^2 x = 0$. Since $x > 0$, we have $\lambda^2 = 4$, which implies $\lambda = 2$ or -2. If $\lambda = 2$ then $y = -x < 0$ as $x > 0$, and this violates the condition $y > 0$. Thus we must have $\lambda = -2$, which implies $y = x$. Plugging this into the third equation above we then have $2x^2 = 1$, and hence $x = y = \sqrt{2}/2$. Since this is the unique stationary point in the domain $x > 0, y > 0$, it must be a maximum point. The maximum area of the rectangle is $4 \times (\sqrt{2}/2) \times (\sqrt{2}/2) = 2$.

9. (a) (i) $z_1 + z_2 = 3 - 2i$

(ii) $2z_1 - 5z_2 = 2(2 - 3i) - 5(1 + i) = (4 - 6i) - (5 + 5i) = -1 - 11i$

(iii) $z_1 z_2^2 = (2 - 3i)(1 + i)^2 = (2 - 3i)(1 + 2i + i^2) = 2i(2 - 3i) = 6 + 4i$

(iv) $\frac{z_1}{z_2} = \frac{2-3i}{1+i} = \frac{(2-3i)(1-i)}{(1+i)(1-i)} = \frac{-1-5i}{2} = -\frac{1}{2} - \frac{5}{2}i$

(b) (i) By completing the square we get $(z+2)^2 = -3$, which gives $z+2 = \pm\sqrt{3}i$. Thus, $z_1 = -2 + \sqrt{3}i, z_1 = -2 - \sqrt{3}i$.

Another method: By the quadratic formula we get $z = \frac{-4 \pm \sqrt{4^2 - 4 \times 7}}{2} = \frac{-4 \pm 2\sqrt{3}i}{2} = -2 \pm \sqrt{3}i$.

(ii) The given equation can be factorized as $z(4z^2 - 4z + 3) = 0$. Thus, $z = 0$ or $4z^2 - 4z + 3 = 0$. For the latter one, by completing the square we have $(2z - 1)^2 = -2$, which gives $2z - 1 = \pm\sqrt{2}i$, that is, $z = \frac{1}{2} \pm \frac{\sqrt{2}}{2}i$. Thus, the solutions are $z_1 = 0, z_2 = \frac{1}{2} + \frac{\sqrt{2}}{2}i, z_3 = \frac{1}{2} - \frac{\sqrt{2}}{2}i$.

Quadratic formula for $4z^2 - 4z + 3 = 0$: $z = \frac{4 \pm \sqrt{(-4)^2 - 4 \times 4 \times 3}}{8} = \frac{4 \pm 4\sqrt{2}i}{8} = \frac{1}{2} \pm \frac{\sqrt{2}}{2}i$.

10. (a) (i) For $y = Cxe^{-\frac{1}{x}}$, we have $\frac{dy}{dx} = C(e^{-\frac{1}{x}} + xe^{-\frac{1}{x}}(\frac{1}{x^2})) = C(1 + \frac{1}{x})e^{-\frac{1}{x}}$. On the other hand $\frac{(x+1)y}{x^2} = \frac{x+1}{x^2} \cdot Cxe^{-\frac{1}{x}} = C(1 + \frac{1}{x})e^{-\frac{1}{x}}$. Therefore, $y = Cxe^{-\frac{1}{x}}$ is the general solution to the given differential equation.

(ii) Since $y(1) = 2$, we have $2 = C \cdot 1 \cdot e^{-\frac{1}{1}} = Ce^{-1}$ and hence $C = 2e$. The particular solution satisfying $y(1) = 2$ is $y = 2xe^{1-\frac{1}{x}}$.

(b) (i) $y = -2e^x + x^5 - \frac{1}{2}x^2 + C$, where $C \in \mathbf{R}$.

(ii) Solving $\frac{dx}{dy} = 1/(y + 1)$ instead we get $x = \int \frac{dy}{y+1} = \log_e |y + 1| + c$, where $c \in \mathbf{R}$. Hence $y + 1 = \pm e^{-c}e^x$, and the general solution is $y = -1 + Ce^x$, where $C \in \mathbf{R}$.