1. (5 marks)

   Simplify

$$\frac{1}{3+\frac{1}{1-\frac{1}{4+\frac{1}{2}}}}$$

2. (5 marks)

   A square island 2km on a side is centred in a circular lake 4km in diameter. Find the shortest distance, in kilometres, from the coast of the island to the mainland.

3. (5 marks)

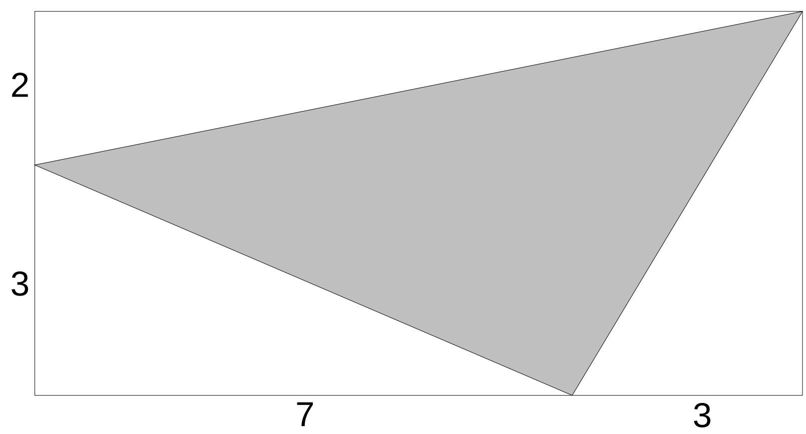
   A farm has cows and chickens. Together they have 35 heads and 94 legs. How many more chickens than cows?

4. (5 marks)

   How many positive factors (including 1 and itself) has the number 210?

5. (5 marks) **(CHANGE RUNNER NOW)**

   A triangle is inscribed in a rectangle as shown. From the lengths given, find the area of the triangle.



6. (5 marks)

   Given

$$6^{2x+y} = 36^7 \quad \text{and} \quad 6^{x+4y} = 216^7,$$

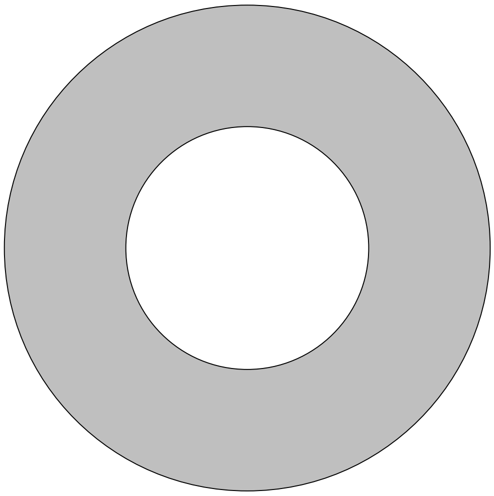
   determine xy.

7. (5 marks)

   How many ways are there of obtaining 50 cents change from a large enough supply of 5, 10 and 20 cent coins?

8. (5 marks)

   The area of the region between the two concentric circles shaded below is 100π. What is the length of the longest line segment that fits within the region?



9. (10 marks)

   Find all y satisfying

$$|y+1| + 2|y-2| < 6$$

10. (10 marks) **(CHANGE RUNNER NOW)**

    Find an integer n such that both $n+17$ and $n-20$ are both perfect squares.

11. (10 marks)

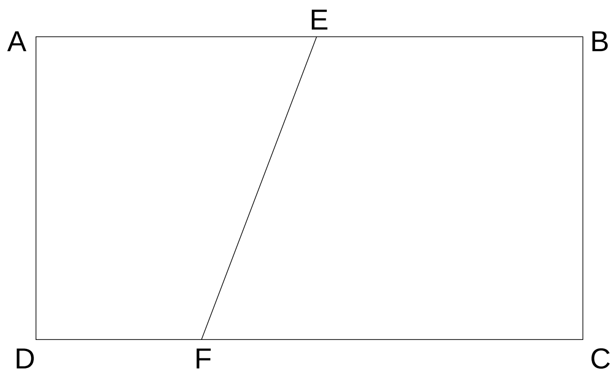
    A sequence of numbers k_1, k_2, k_3, ... satisfies $k_1 = 0$ and

$$k_{n+1} = \frac{n}{n+1}(k_n + 1) \quad \text{for all } n \geq 1.$$

    Find k_{1999}.

12. (10 marks)

Rectangle $ABCD$ is divided into two trapeziums by line segment EF, where E is the mid-point of AB, and F is a point on CD. If the area of trapezium $AEFD$ is half the area of trapezium $EBCF$, find the ratio $DF : FC$.



13. (10 marks)

A rectangular tank with base 4 metres by 3 metres and height 3 metres contains water to a depth of 1 metre. A lead cube of edge length two metres is placed in the tank. By how much (in centimetres) does the depth of water increase?

14. (10 marks)

Two different numbers are randomly chosen from the set $\{1, 2, \ldots, 12\}$. Determine the probability that one of the numbers is a multiple of the other.

(Write your answer in the form $\frac{a}{b}$, where a and b have no common factors.)

15. (10 marks) **(CHANGE RUNNER NOW)**

One mathematician once said, "Today I will prove more theorems than two days ago but fewer than one week ago." What is the greatest number of consecutive days that she could have said this and still be telling the truth?

16. (10 marks)

Two amateurs were allowed to enter a chess tournament otherwise comprising professionals. Each contestant played once with every other contestant and received one point for a win, half a point for a draw, and zero points for a loss. The two amateurs together gained a total of eight points and all professionals scored the same number of points as each other.

Determine the maximum possible number of professionals in the tournament.

17. (15 marks)

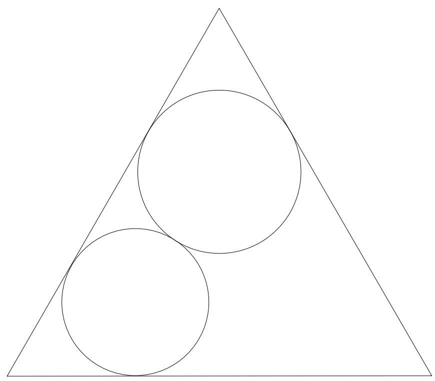
The sum of the cubes of two positive integers is 6293. What is the sum of their squares?

18. (15 marks)

The numbers $1, 2, \ldots, 10$ are placed in order so that each number is either strictly greater than all the preceding numbers or else strictly less than all preceding numbers. In how many ways can this be done?

19. (20 marks)

The triangle below is equilateral with perimeter $33\sqrt{3}$. The circles are tangential to each other and to the triangle as shown. If the radii of the circles differ by 1 unit, what is their sum?



20. (20 marks)

A set of consecutive numbers beginning with 1 is written on a blackboard. One number is erased. The average (arithmetic mean) of the remaining numbers is $35\frac{1}{4}$. What number was erased?